Mathematica 11.3 Integration Test Results

Test results for the 594 problems in "1.1.3.8 P(x) (c x)^m (a+b x^n)^p.m"

Problem 21: Result more than twice size of optimal antiderivative.

$$\int \frac{a^{1/3} (-b)^{1/3} B - (-b)^{2/3} B x}{a + b x^3} \, dx$$

Optimal (type 3, 41 leaves, 3 steps):

$$\frac{2 B \operatorname{ArcTan}\left[\frac{a^{1/3}+2 (-b)^{1/3} x}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{1/3}}$$

Result (type 3, 129 leaves):

$$\frac{1}{6 a^{1/3} b^{2/3}} (-b)^{1/3} B \left(2 \sqrt{3} \left((-b)^{1/3} - b^{1/3}\right) \operatorname{ArcTan}\left[\frac{1 - \frac{2 b^{1/3} x}{a^{1/3}}}{\sqrt{3}}\right] + \left((-b)^{1/3} + b^{1/3}\right) \left(2 \operatorname{Log}\left[a^{1/3} + b^{1/3} x\right] - \operatorname{Log}\left[a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2\right]\right)\right)$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \frac{(-a)^{2/3} C + 2 C x^2}{a - 8 x^3} \, dx$$

Optimal (type 3, 47 leaves, 4 steps):

$$\frac{C \operatorname{ArcTan}\left[\frac{1-\frac{4 x}{(-a)^{1/3}}}{\sqrt{3}}\right]}{2 \sqrt{3}} - \frac{1}{4} C \operatorname{Log}\left[(-a)^{1/3} + 2 x\right]$$

Result (type 3, 106 leaves):

$$\frac{1}{12 a^{2/3}} C \left(2 \sqrt{3} (-a)^{2/3} \operatorname{ArcTan}\left[\frac{1 + \frac{4 x}{a^{1/3}}}{\sqrt{3}}\right] - 2 (-a)^{2/3} \operatorname{Log}\left[a^{1/3} - 2 x\right] + (-a)^{2/3} \operatorname{Log}\left[a^{2/3} + 2 a^{1/3} x + 4 x^2\right] - a^{2/3} \operatorname{Log}\left[-a + 8 x^3\right]\right)$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int \frac{2\left(\frac{a}{b}\right)^{2/3} C + C x^2}{a + b x^3} \, dx$$

Optimal (type 3, 50 leaves, 4 steps):

$$-\frac{2\, C\, \text{ArcTan}\left[\frac{1-\frac{2 x}{\left(\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b}+\frac{C\, \text{Log}\left[\left(\frac{a}{b}\right)^{1/3}+x\right]}{b}$$

Result (type 3, 146 leaves):

$$\frac{1}{3\, a^{2/3}\, b} C \left(-2\sqrt{3}\left(\frac{a}{b}\right)^{2/3} b^{2/3}\, \text{ArcTan}\left[\frac{1-\frac{2\, b^{1/3}\, x}{a^{1/3}}}{\sqrt{3}}\right]+2\left(\frac{a}{b}\right)^{2/3} b^{2/3}\, \text{Log}\left[a^{1/3}+b^{1/3}\, x\right]-\left(\frac{a}{b}\right)^{2/3} b^{2/3}\, \text{Log}\left[a^{2/3}-a^{1/3}\, b^{1/3}\, x+b^{2/3}\, x^2\right]+a^{2/3}\, \text{Log}\left[a+b\, x^3\right]\right)$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3} C + C x^2}{a - b x^3} \, dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$\frac{2\, C\, \text{ArcTan}\left[\frac{1-\frac{2 x}{\left(-\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\, b}-\frac{C\, \text{Log}\left[\left(-\frac{a}{b}\right)^{1/3}+x\right]}{b}$$

Result (type 3, 150 leaves):

$$\frac{1}{3\, a^{2/3}\, b} C \left(2\sqrt{3}\left(-\frac{a}{b}\right)^{2/3} b^{2/3}\, \text{ArcTan}\left[\frac{1+\frac{2\, b^{1/3}\, x}{a^{1/3}}}{\sqrt{3}}\right]-2\left(-\frac{a}{b}\right)^{2/3} b^{2/3}\, \text{Log}\left[a^{1/3}-b^{1/3}\, x\right]+\left(-\frac{a}{b}\right)^{2/3} b^{2/3}\, \text{Log}\left[a^{2/3}+a^{1/3}\, b^{1/3}\, x+b^{2/3}\, x^2\right]-a^{2/3}\, \text{Log}\left[a-b\, x^3\right]\right)$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int \frac{2\left(-\frac{a}{b}\right)^{2/3} C + C x^2}{a + b x^3} \, dx$$

Optimal (type 3, 54 leaves, 4 steps):

$$-\frac{2\,C\,\text{ArcTan}\left[\frac{1+\frac{2\,x}{\left(-\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b}+\frac{C\,\text{Log}\left[\left(-\frac{a}{b}\right)^{1/3}-x\right]}{b}$$

Result (type 3, 149 leaves):

$$\frac{1}{3\,a^{2/3}\,b}C\left(-2\,\sqrt{3}\,\left(-\frac{a}{b}\right)^{2/3}\,b^{2/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,b^{1/3}\,x}{a^{1/3}}}{\sqrt{3}}\right]+2\,\left(-\frac{a}{b}\right)^{2/3}\,b^{2/3}\,\text{Log}\left[a^{1/3}+b^{1/3}\,x\right]-\left(-\frac{a}{b}\right)^{2/3}\,b^{2/3}\,\text{Log}\left[a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2\right]+a^{2/3}\,\text{Log}\left[a+b\,x^3\right]\right)$$

Problem 36: Result more than twice size of optimal antiderivative.

$$\int\frac{2\,\left(\frac{a}{b}\right)^{2/3}\,C+C\,x^2}{a-b\,x^3}\,dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$\frac{2\,C\,\text{ArcTan}\left[\frac{1+\frac{2\,x}{\left(\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b}-\frac{C\,\text{Log}\left[\left(\frac{a}{b}\right)^{1/3}-x\right]}{b}$$

Result (type 3, 147 leaves):

$$\frac{1}{3\,a^{2/3}\,b}C\left(2\,\sqrt{3}\,\left(\frac{a}{b}\right)^{2/3}\,b^{2/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{a^{1/3}}}{\sqrt{3}}\right]-2\,\left(\frac{a}{b}\right)^{2/3}\,b^{2/3}\,\text{Log}\left[a^{1/3}-b^{1/3}\,x\right]+\left(\frac{a}{b}\right)^{2/3}\,b^{2/3}\,\text{Log}\left[a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2\right]-a^{2/3}\,\text{Log}\left[a-b\,x^3\right]\right)$$

Problem 41: Result more than twice size of optimal antiderivative.

$$\int\frac{a^{1/3}\,(-b)^{1/3}\,B-2\,a^{2/3}\,C-(-b)^{2/3}\,B\,x-(-b)^{2/3}\,C\,x^2}{a+b\,x^3}\,dx$$

Optimal (type 3, 88 leaves, 4 steps):

$$\frac{2\,\left(b\,B+a^{1/3}\,(-b)^{2/3}\,C\right)\,\text{ArcTan}\left[\frac{a^{1/3}+2\,(-b)^{1/3}\,x}{\sqrt{3}\,a^{1/3}}\right]}{\sqrt{3}\,a^{1/3}\,b}+\frac{C\,\text{Log}\left[a^{1/3}-(-b)^{1/3}\,x\right]}{(-b)^{1/3}}$$

Result (type 3, 238 leaves):

$$\frac{1}{6\,a^{1/3}\,b}\left(2\,\sqrt{3}\,b^{1/3}\,\left(\left(\left(-b\right)^{2/3}-\left(-b^2\right)^{1/3}\right)\,B+2\,a^{1/3}\,b^{1/3}\,C\right)\,\text{ArcTan}\left[\frac{1-\frac{2\,b^{1/3}\,x}{a^{1/3}}}{\sqrt{3}}\right]+\frac{1}{\left(-b^2\right)^{1/3}}\left(-2\,b\,\left(\left(-\left(-b\right)^{2/3}+b^{2/3}\right)\,B+2\,a^{1/3}\,\left(-b\right)^{1/3}\,C\right)\,\text{Log}\left[a^{1/3}+b^{1/3}\,x\right]+\left(\left(-b\right)^{5/3}\,B+b^{5/3}\,B+2\,a^{1/3}\,\left(-b\right)^{1/3}\,b\,C\right)\,\text{Log}\left[a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2\right]-2\,a^{1/3}\,\left(-b\right)^{2/3}\,\left(-b^2\right)^{1/3}\,C\,\text{Log}\left[a+b\,x^3\right]\right)\right)$$

Problem 44: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(\frac{a}{b}\right)^{1/3}\,B+2\,\left(\frac{a}{b}\right)^{2/3}\,C+B\,x+C\,x^2}{a+b\,x^3}\,dx$$

Optimal (type 3, 71 leaves, 4 steps):

$$-\frac{2\,\left(\frac{a}{b}\right)^{2/3}\,\left(B+\left(\frac{a}{b}\right)^{1/3}\,C\right)\,\text{ArcTan}\left[\frac{1-\frac{2\,x}{\left(\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,a}+\frac{C\,\text{Log}\left[\left(\frac{a}{b}\right)^{1/3}+x\right]}{b}$$

Result (type 3, 247 leaves):

$$\frac{1}{6\,a\,b}\left(2\,\sqrt{3}\,a^{1/3}\,b^{1/3}\,\left(a^{1/3}\,B+\left(\frac{a}{b}\right)^{1/3}\,b^{1/3}\,\left(B+2\,\left(\frac{a}{b}\right)^{1/3}\,C\right)\right)\,\text{ArcTan}\left[\frac{-a^{1/3}+2\,b^{1/3}\,x}{\sqrt{3}\,a^{1/3}}\right]+2\,b^{1/3}\,\left(-a^{2/3}\,B+a^{1/3}\,\left(\frac{a}{b}\right)^{1/3}\,b^{1/3}\,\left(B+2\,\left(\frac{a}{b}\right)^{1/3}\,C\right)\right)\,\text{Log}\left[a^{1/3}+b^{1/3}\,x\right]+b^{1/3}\,\left(a^{2/3}\,B-a^{1/3}\,\left(\frac{a}{b}\right)^{1/3}\,b^{1/3}\,\left(B+2\,\left(\frac{a}{b}\right)^{1/3}\,C\right)\right)\,\text{Log}\left[a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2\right]+2\,a\,C\,\text{Log}\left[a+b\,x^3\right]\right)$$

Problem 45: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(-\frac{a}{b}\right)^{1/3}\,B+2\,\left(-\frac{a}{b}\right)^{2/3}\,C+B\,x+C\,x^2}{a-b\,x^3}\,dx$$

Optimal (type 3, 76 leaves, 4 steps):

$$\frac{2\,\left(B+\left(-\frac{a}{b}\right)^{1/3}\,C\right)\,\text{ArcTan}\left[\frac{1-\frac{2\,x}{\left(-\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,\left(-\frac{a}{b}\right)^{1/3}\,b}-\frac{C\,\text{Log}\left[\left(-\frac{a}{b}\right)^{1/3}+x\right]}{b}$$

Result (type 3, 288 leaves):

$$-\frac{1}{\sqrt{3}\ a\ b^{2/3}}\left(a^{2/3}\ B-a^{1/3}\left(-\frac{a}{b}\right)^{1/3} b^{1/3}\ B-2\ a^{1/3}\left(-\frac{a}{b}\right)^{2/3} b^{1/3}\ C\right) \text{ArcTan}\left[\frac{a^{1/3}+2\ b^{1/3}\ x}{\sqrt{3}\ a^{1/3}}\right]-\frac{\left(a^{2/3}\ B+a^{1/3}\left(-\frac{a}{b}\right)^{1/3} b^{1/3}\ B+2\ a^{1/3}\left(-\frac{a}{b}\right)^{2/3} b^{1/3}\ C\right) \text{Log}\left[a^{1/3}-b^{1/3}\ x\right]}{3\ a\ b^{2/3}}-\frac{1}{6\ a\ b^{2/3}}\left(-a^{2/3}\ B-a^{1/3}\left(-\frac{a}{b}\right)^{1/3} b^{1/3}\ B-2\ a^{1/3}\left(-\frac{a}{b}\right)^{2/3} b^{1/3}\ C\right) \text{Log}\left[a^{2/3}+a^{1/3}\ b^{1/3}\ x+b^{2/3}\ x^2\right]-\frac{C\ \text{Log}\left[a-b\ x^3\right]}{3\ b}$$

Problem 46: Result more than twice size of optimal antiderivative.

$$\int \frac{-\left(-\frac{a}{b}\right)^{1/3} B+2\left(-\frac{a}{b}\right)^{2/3} C+B\ x+C\ x^2}{a+b\ x^3}\,dx$$

Optimal (type 3, 78 leaves, 4 steps):

$$\frac{2\left(B-\left(-\frac{a}{b}\right)^{1/3} C\right) \text{ArcTan}\left[\frac{1+\frac{2\ x}{\left(-\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\left(-\frac{a}{b}\right)^{1/3} b}+\frac{C\ \text{Log}\left[\left(-\frac{a}{b}\right)^{1/3}-x\right]}{b}$$

Result (type 3, 253 leaves):

$$\frac{1}{6\ a\ b}\left(2\sqrt{3}\ a^{1/3}\ b^{1/3}\left(a^{1/3}\ B+\left(-\frac{a}{b}\right)^{1/3} b^{1/3}\left(-B+2\left(-\frac{a}{b}\right)^{1/3} C\right)\right) \text{ArcTan}\left[\frac{-a^{1/3}+2\ b^{1/3}\ x}{\sqrt{3}\ a^{1/3}}\right]-2\ b^{1/3}\left(a^{2/3}\ B+a^{1/3}\left(-\frac{a}{b}\right)^{1/3} b^{1/3}\left(B-2\left(-\frac{a}{b}\right)^{1/3} C\right)\right) \text{Log}\left[a^{1/3}+b^{1/3}\ x\right]+b^{1/3}\left(a^{2/3}\ B+a^{1/3}\left(-\frac{a}{b}\right)^{1/3} b^{1/3}\left(B-2\left(-\frac{a}{b}\right)^{1/3} C\right)\right) \text{Log}\left[a^{2/3}-a^{1/3}\ b^{1/3}\ x+b^{2/3}\ x^2\right]+2\ a\ C\ \text{Log}\left[a+b\ x^3\right]\right)$$

Problem 47: Result more than twice size of optimal antiderivative.

$$\int \frac{-\left(\frac{a}{b}\right)^{1/3} B+2\left(\frac{a}{b}\right)^{2/3} C+B\ x+C\ x^2}{a-b\ x^3}\,dx$$

Optimal (type 3, 75 leaves, 4 steps):

$$-\frac{2\left(\frac{a}{b}\right)^{2/3}\left(B-\left(\frac{a}{b}\right)^{1/3} C\right) \text{ArcTan}\left[\frac{1+\frac{2\ x}{\left(\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\ a}-\frac{C\ \text{Log}\left[\left(\frac{a}{b}\right)^{1/3}-x\right]}{b}$$

Result (type 3, 244 leaves):

$$\frac{1}{6\,a\,b}\left(-2\sqrt{3}\,a^{1/3}\,b^{1/3}\left(a^{1/3}\,B+\left(\frac{a}{b}\right)^{1/3}b^{1/3}\left(B-2\left(\frac{a}{b}\right)^{1/3}C\right)\right)\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{a^{1/3}}}{\sqrt{3}}\right]-\right.$$
$$2\,b^{1/3}\left(a^{2/3}\,B+a^{1/3}\left(\frac{a}{b}\right)^{1/3}b^{1/3}\left(-B+2\left(\frac{a}{b}\right)^{1/3}C\right)\right)\text{Log}\left[a^{1/3}-b^{1/3}\,x\right]+$$
$$\left.b^{1/3}\left(a^{2/3}\,B+a^{1/3}\left(\frac{a}{b}\right)^{1/3}b^{1/3}\left(-B+2\left(\frac{a}{b}\right)^{1/3}C\right)\right)\text{Log}\left[a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2\right]-2\,a\,C\,\text{Log}\left[a-b\,x^3\right]\right)$$

Problem 59: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(a+b\,x^3\right)^{3/2}\left(a\,c+a\,d\,x+b\,c\,x^3+b\,d\,x^4\right)\,dx$$

Optimal (type 4, 585 leaves, 7 steps):

$$\frac{810\,a^3\,d\,\sqrt{a+b\,x^3}}{1729\,b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+\frac{54\,a^2\left(1729\,c\,x+935\,d\,x^2\right)\sqrt{a+b\,x^3}}{323\,323}+$$
$$\frac{30\,a\left(247\,c\,x+187\,d\,x^2\right)\left(a+b\,x^3\right)^{3/2}}{46\,189}+\frac{2}{323}\left(19\,c\,x+17\,d\,x^2\right)\left(a+b\,x^3\right)^{5/2}-$$
$$\left(405\times 3^{1/4}\sqrt{2-\sqrt{3}}\,a^{10/3}\,d\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$
$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$
$$\left(1729\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\sqrt{a+b\,x^3}\right)+$$
$$\left(54\times 3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(1729\,b^{1/3}\,c-935\left(1-\sqrt{3}\right)a^{1/3}\,d\right)\left(a^{1/3}+b^{1/3}\,x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$
$$\left(323\,323\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\sqrt{a+b\,x^3}\right)$$

Result (type 4, 349 leaves):

$$\frac{1}{323323\,(-b)^{2/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\,x\,\left(a+b\,x^3\right)\right.$$
$$\left(1001\,b^2\,x^6\,(19\,c+17\,d\,x)+7\,a\,b\,x^3\,(9139\,c+7667\,d\,x)+a^2\,(91637\,c+61897\,d\,x)\right)-$$
$$151470\,(-1)^{2/3}\,3^{1/4}\,a^{11/3}\,d\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$
$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+$$
$$54\,i\,3^{3/4}\,a^{10/3}\,\left(1729\,(-b)^{1/3}\,c+935\,a^{1/3}\,d\right)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$
$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 60: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{a+b\,x^3}\,\left(a\,c+a\,d\,x+b\,c\,x^3+b\,d\,x^4\right)\,dx$$

Optimal (type 4, 556 leaves, 6 steps):

$$\frac{54\, a^2\, d\, \sqrt{a+b\, x^3}}{91\, b^{2/3}\, \left(\left(1+\sqrt{3}\right)\, a^{1/3}+b^{1/3}\, x\right)}+$$

$$\frac{18\, a\, \left(91\, c\, x+55\, d\, x^2\right)\, \sqrt{a+b\, x^3}}{5005}+\frac{2}{143}\, \left(13\, c\, x+11\, d\, x^2\right)\, \left(a+b\, x^3\right)^{3/2}-$$

$$\left(27\times 3^{1/4}\, \sqrt{2-\sqrt{3}}\, a^{7/3}\, d\, \left(a^{1/3}+b^{1/3}\, x\right)\, \sqrt{\frac{a^{2/3}-a^{1/3}\, b^{1/3}\, x+b^{2/3}\, x^2}{\left(\left(1+\sqrt{3}\right)\, a^{1/3}+b^{1/3}\, x\right)^2}}\right.$$

$$\left.\texttt{EllipticE}\left[\texttt{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\, a^{1/3}+b^{1/3}\, x}{\left(1+\sqrt{3}\right)\, a^{1/3}+b^{1/3}\, x}\right],\, -7-4\, \sqrt{3}\right]\right)\Bigg/$$

$$\left(91\, b^{2/3}\, \sqrt{\frac{a^{1/3}\, \left(a^{1/3}+b^{1/3}\, x\right)}{\left(\left(1+\sqrt{3}\right)\, a^{1/3}+b^{1/3}\, x\right)^2}}\, \sqrt{a+b\, x^3}\right)+$$

$$\left(18\times 3^{3/4}\, \sqrt{2+\sqrt{3}}\, a^2\, \left(91\, b^{1/3}\, c-55\, \left(1-\sqrt{3}\right)\, a^{1/3}\, d\right)\, \left(a^{1/3}+b^{1/3}\, x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\, b^{1/3}\, x+b^{2/3}\, x^2}{\left(\left(1+\sqrt{3}\right)\, a^{1/3}+b^{1/3}\, x\right)^2}}\, \texttt{EllipticF}\left[\texttt{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\, a^{1/3}+b^{1/3}\, x}{\left(1+\sqrt{3}\right)\, a^{1/3}+b^{1/3}\, x}\right],\, -7-4\, \sqrt{3}\right]\right)\Bigg/$$

$$\left(5005\, b^{2/3}\, \sqrt{\frac{a^{1/3}\, \left(a^{1/3}+b^{1/3}\, x\right)}{\left(\left(1+\sqrt{3}\right)\, a^{1/3}+b^{1/3}\, x\right)^2}}\, \sqrt{a+b\, x^3}\right)$$

Result (type 4, 329 leaves) :

$$\frac{1}{5005\,(-b)^{2/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\,x\,\left(a+b\,x^3\right)\,\left(1274\,a\,c+880\,a\,d\,x+455\,b\,c\,x^3+385\,b\,d\,x^4\right)-\right.$$

$$2970\,(-1)^{2/3}\,3^{1/4}\,a^{8/3}\,d\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+$$

$$18\,i\,3^{3/4}\,a^{7/3}\,\left(91\,(-b)^{1/3}\,c+55\,a^{1/3}\,d\right)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 61: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a\,c+a\,d\,x+b\,c\,x^3+b\,d\,x^4}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 525 leaves, 5 steps):

$$\frac{6\,a\,d\,\sqrt{a+b\,x^3}}{7\,b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+\frac{2}{35}\left(7\,c\,x+5\,d\,x^2\right)\sqrt{a+b\,x^3}-$$

$$\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^{4/3}\,d\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(7\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+$$

$$\left(2\times 3^{3/4}\sqrt{2+\sqrt{3}}\;a\left(7\,b^{1/3}\,c-5\left(1-\sqrt{3}\right)a^{1/3}\,d\right)\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(35\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 313 leaves) :

$$\frac{1}{35\,(-b)^{2/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\,x\,(7\,c+5\,d\,x)\,\left(a+b\,x^3\right)-30\,(-1)^{2/3}\,3^{1/4}\,a^{5/3}\,d\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\right.$$

$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+$$

$$2\,i\,3^{3/4}\,a^{4/3}\,\left(7\,(-b)^{1/3}\,c+5\,a^{1/3}\,d\right)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 62: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a\,c+a\,d\,x+b\,c\,x^3+b\,d\,x^4}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 490 leaves, 4 steps):

$$\frac{2\,d\,\sqrt{a+b\,x^3}}{b^{2/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-$$

$$\left(3^{1/4}\,\sqrt{2-\sqrt{3}}\,\,a^{1/3}\,d\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticE}\left[\right.\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Big/\left(b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)+$$

$$\left(2\,\sqrt{2+\sqrt{3}}\,\,\left(b^{1/3}\,c-\left(1-\sqrt{3}\right)\,a^{1/3}\,d\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Big/$$

$$\left(3^{1/4}\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 221 leaves):

$$-\left(\left(2\,a^{1/3}\,\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\right.\right.$$

$$\left((-1)^{2/3}\,\sqrt{3}\,\,a^{1/3}\,d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-\right.$$

$$i\,\left((-b)^{1/3}\,c+a^{1/3}\,d\right)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\right.$$

$$\left.\left.\left.\left.(-1)^{1/3}\right]\right)\right)\Big/\left(3^{1/4}\,(-b)^{2/3}\,\sqrt{a+b\,x^3}\right)\right)$$

Problem 63: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a\,c + a\,d\,x + b\,c\,x^3 + b\,d\,x^4}{\left(a + b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 522 leaves, 5 steps):

$$\frac{2\,x\,(c + d\,x)}{3\,a\,\sqrt{a + b\,x^3}} - \frac{2\,d\,\sqrt{a + b\,x^3}}{3\,a\,b^{2/3}\,\left(\left(1 + \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x\right)} +$$

$$\left(\sqrt{2 - \sqrt{3}}\,d\,\left(a^{1/3} + b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3} - a^{1/3}\,b^{1/3}\,x + b^{2/3}\,x^2}{\left(\left(1 + \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x}{\left(1 + \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x}\right],\,-7 - 4\,\sqrt{3}\right]\right) \Big/$$

$$\left(3^{3/4}\,a^{2/3}\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3} + b^{1/3}\,x\right)}{\left(\left(1 + \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x\right)^2}}\,\sqrt{a + b\,x^3}\right) +$$

$$\left(2\,\sqrt{2 + \sqrt{3}}\,\left(b^{1/3}\,c + \left(1 - \sqrt{3}\right)\,a^{1/3}\,d\right)\,\left(a^{1/3} + b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3} - a^{1/3}\,b^{1/3}\,x + b^{2/3}\,x^2}{\left(\left(1 + \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x}{\left(1 + \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x}\right],\,-7 - 4\,\sqrt{3}\right]\right) \Big/$$

$$\left(3 \times 3^{1/4}\,a\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3} + b^{1/3}\,x\right)}{\left(\left(1 + \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x\right)^2}}\,\sqrt{a + b\,x^3}\right)$$

Result (type 4, 305 leaves):

$$\left(6\,(-b)^{2/3}\,x\,(c+d\,x)+6\,(-1)^{2/3}\,3^{1/4}\,a^{2/3}\,d\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\right.$$

$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+$$

$$2\,i\,3^{3/4}\,a^{1/3}\,\left((-b)^{1/3}\,c-a^{1/3}\,d\right)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\Bigg/\left(9\,a\,(-b)^{2/3}\,\sqrt{a+b\,x^3}\right)$$

Problem 64: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a\,c+a\,d\,x+b\,c\,x^3+b\,d\,x^4}{\left(a+b\,x^3\right)^{7/2}}\,dx$$

Optimal (type 4, 554 leaves, 6 steps) :

$$\frac{2x(c+dx)}{9a(a+bx^3)^{3/2}}+\frac{2x(7c+5dx)}{27a^2\sqrt{a+bx^3}}-\frac{10d\sqrt{a+bx^3}}{27a^2b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}+$$

$$\left(5\sqrt{2-\sqrt{3}}\,d\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(9\times 3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)+$$

$$\left(2\sqrt{2+\sqrt{3}}\left(7b^{1/3}c+5\left(1-\sqrt{3}\right)a^{1/3}d\right)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(27\times 3^{1/4}a^2b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)$$

Result (type 4, 267 leaves):

$$\left(2\left(3(-b)^{2/3}\left(2ax(5c+4dx)+bx^4(7c+5dx)\right)+\right.\right.$$

$$3^{3/4}\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}x}{a^{1/3}}\right)}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\left(a+bx^3\right)\left(5(-1)^{2/3}\sqrt{3}\,a^{2/3}d\right.$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+i\,a^{1/3}\left(7(-b)^{1/3}c-5a^{1/3}d\right)$$

$$\left.\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\right)\Big/\left(81a^2(-b)^{2/3}\left(a+bx^3\right)^{3/2}\right)$$

Problem 65: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a\,c + a\,d\,x + b\,c\,x^3 + b\,d\,x^4}{\left(a + b\,x^3\right)^{9/2}}\,dx$$

Optimal (type 4, 581 leaves, 7 steps):

$$\frac{2\,x\,\left(c + d\,x\right)}{15\,a\,\left(a + b\,x^3\right)^{5/2}} + \frac{2\,x\,\left(13\,c + 11\,d\,x\right)}{135\,a^2\,\left(a + b\,x^3\right)^{3/2}} + \frac{2\,x\,\left(91\,c + 55\,d\,x\right)}{405\,a^3\,\sqrt{a + b\,x^3}} - \frac{22\,d\,\sqrt{a + b\,x^3}}{81\,a^3\,b^{2/3}\,\left(\left(1 + \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x\right)} + \left(11\,\sqrt{2 - \sqrt{3}}\,\,d\,\left(a^{1/3} + b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3} - a^{1/3}\,b^{1/3}\,x + b^{2/3}\,x^2}{\left(\left(1 + \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x\right)^2}}\,\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x}{\left(1 + \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x}\right],\,-7 - 4\,\sqrt{3}\,\right]\right)\Big/\left(27 \times 3^{3/4}\,a^{8/3}\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3} + b^{1/3}\,x\right)}{\left(\left(1 + \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x\right)^2}}\,\,\sqrt{a + b\,x^3}\right) + \left(2\,\sqrt{2 + \sqrt{3}}\,\,\left(91\,b^{1/3}\,c + 55\,\left(1 - \sqrt{3}\right)\,a^{1/3}\,d\right)\,\left(a^{1/3} + b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3} - a^{1/3}\,b^{1/3}\,x + b^{2/3}\,x^2}{\left(\left(1 + \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x\right)^2}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x}{\left(1 + \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x}\right],\,-7 - 4\,\sqrt{3}\,\right]\right)\Big/\left(405 \times 3^{1/4}\,a^3\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3} + b^{1/3}\,x\right)}{\left(\left(1 + \sqrt{3}\right)\,a^{1/3} + b^{1/3}\,x\right)^2}}\,\,\sqrt{a + b\,x^3}\right)$$

Result (type 4, 287 leaves):

$$\frac{1}{1215\, a^3\, (-b)^{2/3}\, \left(a+b\, x^3\right)^{5/2}} \left(2 \left(3\, (-b)^{2/3} \left(13\, a\, b\, x^4\, (17\, c+11\, d\, x)+b^2\, x^7\, (91\, c+55\, d\, x)+a^2\, x\, (157\, c+115\, d\, x)\right)+\right.\right.$$

$$3^{3/4} \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3}\, x}{a^{1/3}}\right)} \sqrt{1+\frac{(-b)^{1/3}\, x}{a^{1/3}}+\frac{(-b)^{2/3}\, x^2}{a^{2/3}}} \left(a+b\, x^3\right)^2$$

$$\left(55\, (-1)^{2/3} \sqrt{3}\, a^{2/3}\, d\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\, (-b)^{1/3}\, x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]+\right.$$

$$\left.\left.\left. i\, a^{1/3} \left(91\, (-b)^{1/3}\, c-55\, a^{1/3}\, d\right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\, (-b)^{1/3}\, x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)\right)\right)$$

Problem 66: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c+d\, x+e\, x^2+f\, x^3+g\, x^4}{\sqrt{a+b\, x^3}}\, dx$$

Optimal (type 4, 590 leaves, 7 steps):

$$\frac{2\,e\,\sqrt{a+b\,x^3}}{3\,b}+\frac{2\,f\,x\,\sqrt{a+b\,x^3}}{5\,b}+\frac{2\,g\,x^2\,\sqrt{a+b\,x^3}}{7\,b}+$$

$$\frac{2\,\left(7\,b\,d-4\,a\,g\right)\,\sqrt{a+b\,x^3}}{7\,b^{5/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-\left(3^{1/4}\,\sqrt{2-\sqrt{3}}\,\,a^{1/3}\,\left(7\,b\,d-4\,a\,g\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\bigg/$$

$$\left(7\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)+$$

$$\left(2\,\sqrt{2+\sqrt{3}}\,\,\left(7\,b^{1/3}\,\left(5\,b\,c-2\,a\,f\right)-5\,\left(1-\sqrt{3}\right)\,a^{1/3}\,\left(7\,b\,d-4\,a\,g\right)\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\bigg/$$

$$\left(35\times 3^{1/4}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 357 leaves) :

$$-\frac{1}{105\,(-b)^{5/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\,(a+b\,x^3)\,(35\,e+3\,x\,(7\,f+5\,g\,x))-\right.$$

$$30\,(-1)^{2/3}\,3^{1/4}\,a^{2/3}\,(7\,b\,d-4\,a\,g)\,\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+2\,i\,3^{3/4}\,a^{1/3}$$

$$\left(35\,b\,\left((-b)^{1/3}\,c+a^{1/3}\,d\right)-2\,a\,\left(7\,(-b)^{1/3}\,f+10\,a^{1/3}\,g\right)\right)\,\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 67: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x+e\,x^2+f\,x^3+g\,x^4}{(a+b\,x^3)^{3/2}}\,dx$$

Optimal (type 4, 594 leaves, 6 steps):

$$\frac{2 x \left(b c - a f + \left(b d - a g\right) x + b e x^2\right)}{3 a b \sqrt{a + b x^3}} - \frac{2 e \sqrt{a + b x^3}}{3 a b} - \frac{2 \left(b d - 4 a g\right) \sqrt{a + b x^3}}{3 a b^{5/3} \left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)} + \left(\sqrt{2 - \sqrt{3}} \left(b d - 4 a g\right) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Bigg/ \left(3^{3/4} a^{2/3} b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right) + \left(2 \sqrt{2 + \sqrt{3}} \left(b^{1/3} \left(b c + 2 a f\right) + \left(1 - \sqrt{3}\right) a^{1/3} \left(b d - 4 a g\right)\right) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Bigg/ \left(3 \times 3^{1/4} a b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 354 leaves) :

$$-\frac{1}{9\,a\,(-b)^{5/3}\,\sqrt{a+b\,x^3}}\left(6\,(-b)^{2/3}\,\left(b\,x\,(c+d\,x)-a\,(e+x\,(f+g\,x))\right)+\right.$$

$$6\,(-1)^{2/3}\,3^{1/4}\,a^{2/3}\,(b\,d-4\,a\,g)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+$$

$$2\,i\,3^{3/4}\,a^{1/3}\,\left((-b)^{1/3}\,b\,c-a^{1/3}\,b\,d+2\,a\,(-b)^{1/3}\,f+4\,a^{4/3}\,g\right)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 68: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x+e\,x^2+f\,x^3+g\,x^4}{\left(a+b\,x^3\right)^{5/2}}\,dx$$

Optimal (type 4, 628 leaves, 5 steps):

$$\frac{2 x \left(b c - a f + \left(b d - a g\right) x + b e x^2\right)}{9 a b \left(a + b x^3\right)^{3/2}} - \frac{2 \left(5 b d + 4 a g\right) \sqrt{a + b x^3}}{27 a^2 b^{5/3} \left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)} - \frac{2 \left(3 a e - x \left(7 b c + 2 a f + \left(5 b d + 4 a g\right) x\right)\right)}{27 a^2 b \sqrt{a + b x^3}} +$$

$$\left(\sqrt{2 - \sqrt{3}} \left(5 b d + 4 a g\right) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/$$

$$\left(9 \times 3^{3/4} a^{5/3} b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right) +$$

$$\left(2 \sqrt{2 + \sqrt{3}} \left(b^{1/3} \left(7 b c + 2 a f\right) + \left(1 - \sqrt{3}\right) a^{1/3} \left(5 b d + 4 a g\right)\right) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Big/$$

$$\left(27 \times 3^{1/4} a^2 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 329 leaves):

$$-\frac{1}{81\,a^2\,(-b)^{5/3}\,\left(a+b\,x^3\right)^{3/2}}2\left(-3\,(-b)^{2/3}\left(-x\,\left(7\,b\,c+2\,a\,f+5\,b\,d\,x+4\,a\,g\,x\right)\,\left(a+b\,x^3\right)+3\,a\,\left(-b\,x\,\left(c+d\,x\right)+a\,\left(e+x\,\left(f+g\,x\right)\right)\right)\right)+\right.$$

$$i\,3^{3/4}\,a^{1/3}\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\left(a+b\,x^3\right)$$

$$\left((-1)^{1/6}\,\sqrt{3}\,a^{1/3}\,\left(5\,b\,d+4\,a\,g\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+\right.$$

$$\left(7\,(-b)^{1/3}\,b\,c-5\,a^{1/3}\,b\,d+2\,a\,(-b)^{1/3}\,f-4\,a^{4/3}\,g\right)$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)$$

Problem 69: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x+e\,x^2+f\,x^3+g\,x^4}{\left(a+b\,x^3\right)^{7/2}}\,dx$$

Optimal (type 4, 676 leaves, 6 steps):

$$\frac{2\,x\,\left(b\,c-a\,f+\left(b\,d-a\,g\right)\,x+b\,e\,x^2\right)}{15\,a\,b\,\left(a+b\,x^3\right)^{5/2}}+\frac{2\,x\,\left(7\,\left(13\,b\,c+2\,a\,f\right)+5\,\left(11\,b\,d+4\,a\,g\right)\,x\right)}{405\,a^3\,b\,\sqrt{a+b\,x^3}}-\frac{2\,\left(11\,b\,d+4\,a\,g\right)\,\sqrt{a+b\,x^3}}{81\,a^3\,b^{5/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-\frac{2\,\left(9\,a\,e-x\,\left(13\,b\,c+2\,a\,f+\left(11\,b\,d+4\,a\,g\right)\,x\right)\right)}{135\,a^2\,b\,\left(a+b\,x^3\right)^{3/2}}+\left(\sqrt{2-\sqrt{3}}\,\left(11\,b\,d+4\,a\,g\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Big/\left(27\times 3^{3/4}\,a^{8/3}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+\left(2\,\sqrt{2+\sqrt{3}}\,\left(7\,b^{1/3}\,\left(13\,b\,c+2\,a\,f\right)+5\,\left(1-\sqrt{3}\right)\,a^{1/3}\,\left(11\,b\,d+4\,a\,g\right)\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Big/\left(405\times 3^{1/4}\,a^3\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 366 leaves) :

$$-\frac{1}{1215\, a^3\, (-b)^{5/3}\, \left(a+b\, x^3\right)^{5/2}}$$

$$2 \left(-3\, (-b)^{2/3} \left(-3\, a\, x\, (13\, b\, c+2\, a\, f+11\, b\, d\, x+4\, a\, g\, x)\, \left(a+b\, x^3\right)-x\, (91\, b\, c+14\, a\, f+ 55\, b\, d\, x+20\, a\, g\, x)\, \left(a+b\, x^3\right)^2+27\, a^2\, (-b\, x\, (c+d\, x)+a\, (e+x\, (f+g\, x)))\right)+\right.$$

$$\mathbb{i}\, 3^{3/4}\, a^{1/3} \sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\, x\right)}{a^{1/3}}}\, \sqrt{1+\frac{(-b)^{1/3}\, x}{a^{1/3}}+\frac{(-b)^{2/3}\, x^2}{a^{2/3}}}\, \left(a+b\, x^3\right)^2$$

$$\left(5\, (-1)^{1/6} \sqrt{3}\, a^{1/3}\, (11\, b\, d+4\, a\, g)\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathbb{i}\, (-b)^{1/3}\, x}{a^{1/3}}}}{3^{1/4}}\right],\, (-1)^{1/3}\right]+\right.$$

$$\left(91\, (-b)^{1/3}\, b\, c-55\, a^{1/3}\, b\, d+14\, a\, (-b)^{1/3}\, f-20\, a^{4/3}\, g\right)$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathbb{i}\, (-b)^{1/3}\, x}{a^{1/3}}}}{3^{1/4}}\right],\, (-1)^{1/3}\right]\right)\right)$$

Problem 79: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\, dx$$

Optimal (type 4, 230 leaves, 3 steps):

$$\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}\right)+$$

$$\left(4\times 3^{1/4}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}\right)$$

Result (type 4, 127 leaves):

$$\frac{1}{\sqrt{1+x^3}}3^{1/4}\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}\,\sqrt{1+(-1)^{1/3}x+(-1)^{2/3}x^2}$$

$$\left(-2\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left.(-1)^{1/6}\left((2-i)+\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 80: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$$

Optimal (type 4, 257 leaves, 3 steps):

$$-\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}+\left(3^{1/4}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}\right)-\left(4\times 3^{1/4}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}\right)$$

Result (type 4, 112 leaves):

$$\frac{1}{\sqrt{1-x^3}}2\times 3^{1/4}\sqrt{(-1)^{5/6}(-1+x)}\,\sqrt{1+x+x^2}\left((-1)^{2/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right],(-1)^{1/3}\right]+i\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 81: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$$

Optimal (type 4, 144 leaves, 1 step):

$$\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}-\left(3^{1/4}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]\right)\Big/\left(\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}\right)$$

Result (type 4, 110 leaves):

$$\frac{1}{\sqrt{-1+x^3}}2\times 3^{1/4}\sqrt{(-1)^{5/6}(-1+x)}\sqrt{1+x+x^2}\left((-1)^{2/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]+i\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 82: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 135 leaves, 1 step):

$$-\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\left(3^{1/4}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]\right)\Big/\left(\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}\right)$$

Result (type 4, 147 leaves):

$$\frac{1}{\sqrt{-1-x^3}}(1-i)(-1)^{1/6}3^{1/4}\sqrt{-(-1)^{5/6}+i\,x}\sqrt{1-(-1)^{2/3}x-(-1)^{1/3}x^2}\left((1+i)(-1)^{1/6}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]-\left(1+\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 83: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\sqrt{a+b x^3}} \, dx$$

Optimal (type 4, 468 leaves, 3 steps):

$$\frac{2 \sqrt{a+b x^3}}{b^{1/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\left(3^{1/4} \sqrt{2-\sqrt{3}}\, a^{1/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/ \left(b^{1/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+\left(4\times 3^{1/4} \sqrt{2+\sqrt{3}}\, a^{1/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/ \left(b^{1/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 225 leaves):

$$\left(2\,\mathrm{i}\,a^{2/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\right.$$

$$\left(-3\,(-1)^{1/6}\,b^{1/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left(\left(3+\sqrt{3}\right)(-b)^{1/3}+\sqrt{3}\,b^{1/3}\right)$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(3^{3/4}\,(-b)^{2/3}\sqrt{a+b\,x^3}\right)$$

Problem 84: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\sqrt{a-b\,x^3}}\,dx$$

Optimal (type 4, 481 leaves, 3 steps):

$$-\frac{2\sqrt{a-b\,x^3}}{b^{1/3}\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)}+$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\text{EllipticE}\left[\right.\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\sqrt{a-b\,x^3}\right)-$$

$$\left(4\times3^{1/4}\sqrt{2+\sqrt{3}}\;a^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\sqrt{a-b\,x^3}\right)$$

Result (type 4, 182 leaves):

$$\frac{1}{b^{1/3}\sqrt{a-b x^3}}2\times 3^{1/4} a^{2/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+b^{1/3} x\right)}{a^{1/3}}}\sqrt{1+\frac{b^{1/3} x}{a^{1/3}}+\frac{b^{2/3} x^2}{a^{2/3}}}$$
$$\left((-1)^{2/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\, b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$
$$\left.i\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\, b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 85: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3} x}{\sqrt{-a+b x^3}}\,dx$$

Optimal (type 4, 271 leaves, 1 step):

$$\frac{2\sqrt{-a+b x^3}}{b^{1/3}\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3} x\right)}-\left(3^{1/4}\sqrt{2+\sqrt{3}}\;a^{1/3}\left(a^{1/3}-b^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}+a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3} x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3} x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3} x}\right],-7+4\sqrt{3}\right]\right)\Bigg/$$
$$\left(b^{1/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3} x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3} x\right)^2}}\;\sqrt{-a+b x^3}\right)$$

Result (type 4, 257 leaves):

$$\left(2\,(-a)^{1/3}\sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}(-b)^{1/3}x\right)}{a}}\sqrt{1+\frac{(-b)^{1/3}x\left((-a)^{1/3}+(-b)^{1/3}x\right)}{(-a)^{2/3}}}\right.$$

$$\left(3\,(-1)^{2/3}(-a)^{1/3}b^{1/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$i\left(\left(3+\sqrt{3}\right)a^{1/3}(-b)^{1/3}-\sqrt{3}\,(-a)^{1/3}b^{1/3}\right)$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(3^{3/4}(-b)^{2/3}\sqrt{-a+b\,x^3}\right)$$

Problem 86: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\sqrt{-a-b\,x^3}}\,dx$$

Optimal (type 4, 266 leaves, 1 step):

$$-\frac{2\sqrt{-a-b\,x^3}}{b^{1/3}\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}+\left(3^{1/4}\sqrt{2+\sqrt{3}}\,a^{1/3}\left(a^{1/3}+b^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{1/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{-a-b\,x^3}\right)$$

Result (type 4, 227 leaves):

$$\left(2\,\mathrm{i}\,(-a)^{1/3}\sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}b^{1/3}x\right)}{a}}\sqrt{1+\frac{b^{1/3}x\left((-a)^{1/3}+b^{1/3}x\right)}{(-a)^{2/3}}}\right.$$

$$\left(-3\,(-1)^{1/6}(-a)^{1/3}\,\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,b^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left(\sqrt{3}\,(-a)^{1/3}+\left(3+\sqrt{3}\right)a^{1/3}\right)$$

$$\left.\left.\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,b^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(3^{3/4}\,b^{1/3}\sqrt{-a-b\,x^3}\right)$$

Problem 87: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1+\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 520 leaves, 3 steps):

$$\frac{2\left(\frac{b}{a}\right)^{1/3}\sqrt{a+b\,x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\left(\frac{b}{a}\right)^{1/3}\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\mathrm{EllipticE}\left[\right.\right.$$

$$\left.\left.\mathrm{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b\,x^3}\right)+$$

$$\left(2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)b^{1/3}-\left(1-\sqrt{3}\right)a^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\right.$$

$$\left.\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4}\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b\,x^3}\right)$$

Result (type 4, 243 leaves) :

$$\left(2\,i\,a^{1/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\right.$$

$$\left(-3\,(-1)^{1/6}a^{1/3}\left(\frac{b}{a}\right)^{1/3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left(\left(3+\sqrt{3}\right)(-b)^{1/3}+\sqrt{3}\,a^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\bigg/\left(3^{3/4}(-b)^{2/3}\sqrt{a+b\,x^3}\right)$$

Problem 88: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1+\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x}{\sqrt{a-b\,x^3}}\,dx$$

Optimal (type 4, 533 leaves, 3 steps) :

$$-\frac{2\left(\frac{b}{a}\right)^{1/3}\sqrt{a-b x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)}+$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\left(\frac{b}{a}\right)^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\sqrt{a-b x^3}\right)-$$

$$\left(2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)b^{1/3}-\left(1-\sqrt{3}\right)a^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)\left(a^{1/3}-b^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4}b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\sqrt{a-b x^3}\right)$$

Result (type 4, 232 leaves):

$$\left(2a^{1/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+b^{1/3}x\right)}{a^{1/3}}}\sqrt{1+\frac{b^{1/3}x}{a^{1/3}}+\frac{b^{2/3}x^2}{a^{2/3}}}\right.$$

$$\left(3(-1)^{2/3}a^{1/3}\left(\frac{b}{a}\right)^{1/3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$i\left(\left(3+\sqrt{3}\right)b^{1/3}-\sqrt{3}\,a^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(3^{3/4}b^{2/3}\sqrt{a-b x^3}\right)$$

Problem 89: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+\sqrt{3}-\left(\frac{b}{a}\right)^{1/3} x}{\sqrt{-a+b x^3}} \, dx$$

Optimal (type 4, 256 leaves, 1 step):

$$\frac{2 \left(\frac{b}{a}\right)^{2/3} \sqrt{-a+b x^3}}{b \left(1-\sqrt{3}-\left(\frac{b}{a}\right)^{1/3} x\right)}-\left(3^{1/4} \sqrt{2+\sqrt{3}} \left(1-\left(\frac{b}{a}\right)^{1/3} x\right) \sqrt{\frac{1+\left(\frac{b}{a}\right)^{1/3} x+\left(\frac{b}{a}\right)^{2/3} x^2}{\left(1-\sqrt{3}-\left(\frac{b}{a}\right)^{1/3} x\right)^2}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-\left(\frac{b}{a}\right)^{1/3} x}{1-\sqrt{3}-\left(\frac{b}{a}\right)^{1/3} x}\right],-7+4 \sqrt{3}\right]\right) \Big/ \left(\left(\frac{b}{a}\right)^{1/3} \sqrt{-\frac{1-\left(\frac{b}{a}\right)^{1/3} x}{\left(1-\sqrt{3}-\left(\frac{b}{a}\right)^{1/3} x\right)^2}} \sqrt{-a+b x^3}\right)$$

Result (type 4, 267 leaves):

$$\left(2 (-a)^{1/3} \sqrt{-\frac{(-1)^{5/6} \left(a+(-a)^{2/3} (-b)^{1/3} x\right)}{a}} \sqrt{1+\frac{(-b)^{1/3} x \left((-a)^{1/3}+(-b)^{1/3} x\right)}{(-a)^{2/3}}} \left(3 (-1)^{2/3} (-a)^{1/3} \left(\frac{b}{a}\right)^{1/3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+i \left(3 (-b)^{1/3}+\sqrt{3} (-b)^{1/3}-\sqrt{3} (-a)^{1/3} \left(\frac{b}{a}\right)^{1/3}\right) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right) \Big/ \left(3^{3/4} (-b)^{2/3} \sqrt{-a+b x^3}\right)$$

Problem 90: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+\sqrt{3}+\left(\frac{b}{a}\right)^{1/3} x}{\sqrt{-a-b x^3}} \, dx$$

Optimal (type 4, 251 leaves, 1 step):

$$-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-b x^3}}{b\left(1-\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x\right)}+$$

$$\left(3^{1/4}\sqrt{2+\sqrt{3}}\left(1+\left(\frac{b}{a}\right)^{1/3}x\right)\sqrt{\frac{1-\left(\frac{b}{a}\right)^{1/3}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1-\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x}{1-\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x}\right],\right.\right.$$

$$\left.\left.-7+4\sqrt{3}\right]\right)\Bigg/\left(\left(\frac{b}{a}\right)^{1/3}\sqrt{-\frac{1+\left(\frac{b}{a}\right)^{1/3}x}{\left(1-\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x\right)^2}}\sqrt{-a-b x^3}\right)$$

Result (type 4, 245 leaves):

$$\left(2\,\mathrm{i}\,(-a)^{1/3}\sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}b^{1/3}x\right)}{a}}\sqrt{1+\frac{b^{1/3}x\left((-a)^{1/3}+b^{1/3}x\right)}{(-a)^{2/3}}}\right.$$

$$\left(-3\,(-1)^{1/6}(-a)^{1/3}\left(\frac{b}{a}\right)^{1/3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,b^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left(\left(3+\sqrt{3}\right)b^{1/3}+\sqrt{3}\,(-a)^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,b^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Bigg/\left(3^{3/4}b^{2/3}\sqrt{-a-b x^3}\right)$$

Problem 91: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$$

Optimal (type 4, 127 leaves, 1 step):

$$\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x}-\left(3^{1/4}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}\right)$$

Result (type 4, 127 leaves) :

$$\frac{1}{\sqrt{1+x^3}}3^{1/4}\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}\,\sqrt{1+(-1)^{1/3}x+(-1)^{2/3}x^2}\left(-2\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/6}\left((-2-\mathrm{i})+\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 92: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$$

Optimal (type 4, 142 leaves, 1 step) :

$$-\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}+\left(3^{1/4}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}\right)$$

Result (type 4, 112 leaves) :

$$\frac{1}{\sqrt{1-x^3}} 2\times 3^{1/4}\sqrt{(-1)^{5/6}(-1+x)}\ \sqrt{1+x+x^2}\left((-1)^{2/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\mathrm{i}\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]-\mathrm{i}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\mathrm{i}\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 93: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$$

Optimal (type 4, 264 leaves, 3 steps):

$$\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}-\left(3^{1/4}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]\right)\Bigg/\left(\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}\right)+\left(4\times 3^{1/4}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]\right)\Bigg/\left(\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}\right)$$

Result (type 4, 110 leaves):

$$\frac{1}{\sqrt{-1+x^3}} 2\times 3^{1/4}\sqrt{(-1)^{5/6}(-1+x)}\ \sqrt{1+x+x^2}\left((-1)^{2/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\mathrm{i}\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]-\mathrm{i}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\mathrm{i}\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 94: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 247 leaves, 3 steps):

$$-\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\left(3^{1/4}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]\right)\Big/\left(\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}\right)-\left(4\times3^{1/4}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]\right)\Big/\left(\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}\right)$$

Result (type 4, 147 leaves):

$$\frac{1}{\sqrt{-1-x^3}}(1+i)\,(-1)^{1/6}\,3^{1/4}\sqrt{-(-1)^{5/6}+i\,x}\,\sqrt{1-(-1)^{2/3}x-(-1)^{1/3}x^2}\left((1-i)\,(-1)^{1/6}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]-\left(-1+\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 95: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx$$

Optimal (type 4, 126 leaves, 1 step):

$$-\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x}+\left(3^{1/4}\sqrt{2-\sqrt{3}}\ (1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\ \sqrt{1+x^3}\right)$$

Result (type 4, 129 leaves) :

$$\frac{1}{\sqrt{1+x^3}}3^{1/4}\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}\ \sqrt{1+(-1)^{1/3}x+(-1)^{2/3}x^2}\left(2\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/6}\left((2+i)-\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 96: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx$$

Optimal (type 4, 143 leaves, 1 step) :

$$\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}-\left(3^{1/4}\sqrt{2-\sqrt{3}}\ (1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\ \sqrt{1-x^3}\right)$$

Result (type 4, 112 leaves) :

$$-\frac{1}{\sqrt{1-x^3}} 2\times 3^{1/4} \sqrt{(-1)^{5/6}(-1+x)} \sqrt{1+x+x^2} \left((-1)^{2/3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right], (-1)^{1/3}\right] - i \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

Problem 97: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}} \, dx$$

Optimal (type 4, 263 leaves, 3 steps):

$$-\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x} + \left(3^{1/4}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]\right) \Big/ \left(\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}\right) - \left(4\times 3^{1/4}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right], -7+4\sqrt{3}\right]\right) \Big/ \left(\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}\right)$$

Result (type 4, 110 leaves):

$$-\frac{1}{\sqrt{-1+x^3}} 2\times 3^{1/4} \sqrt{(-1)^{5/6}(-1+x)} \sqrt{1+x+x^2} \left((-1)^{2/3} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right], (-1)^{1/3}\right] - i \, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-i x}}{3^{1/4}}\right], (-1)^{1/3}\right]\right)$$

Problem 98: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 248 leaves, 3 steps):

$$\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}-\left(3^{1/4}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]\right)\Bigg/\left(\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}\right)+\left(4\times 3^{1/4}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],-7+4\sqrt{3}\right]\right)\Bigg/\left(\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}\right)$$

Result (type 4, 146 leaves):

$$\frac{1}{\sqrt{-1-x^3}}(1+i)\,(-1)^{1/6}\,3^{1/4}\sqrt{-(-1)^{5/6}+i\,x}\,\sqrt{1-(-1)^{2/3}x-(-1)^{1/3}x^2}\left((-1+i)\,(-1)^{1/6}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]+\left(-1+\sqrt{3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 99: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 256 leaves, 1 step):

$$\frac{2\sqrt{a+b x^3}}{b^{1/3}\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\left(3^{1/4}\sqrt{2-\sqrt{3}}\ a^{1/3}\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Bigg/\left(b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \sqrt{a+b x^3}\right)$$

Result (type 4, 225 leaves):

$$\left(2\, i\, a^{2/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3} x\right)}{a^{1/3}}}\ \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\left(-3\,(-1)^{1/6} b^{1/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\left(\left(-3+\sqrt{3}\right)(-b)^{1/3}+\sqrt{3}\ b^{1/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Bigg/\left(3^{3/4}\,(-b)^{2/3}\sqrt{a+b x^3}\right)$$

Problem 100: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(1-\sqrt{3}\right) a^{1/3}-b^{1/3} x}{\sqrt{a-b x^3}}\,dx$$

Optimal (type 4, 263 leaves, 1 step):

$$-\frac{2\sqrt{a-b x^3}}{b^{1/3}\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)}+$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\text{EllipticE}\Big[\right.$$

$$\left.\text{ArcSin}\Big[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}\Big],\,-7-4\sqrt{3}\Big]\right)\Bigg/\left(b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\sqrt{a-b x^3}\right)$$

Result (type 4, 182 leaves):

$$\frac{1}{b^{1/3}\sqrt{a-b x^3}}2\times 3^{1/4}a^{2/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+b^{1/3}x\right)}{a^{1/3}}}\sqrt{1+\frac{b^{1/3}x}{a^{1/3}}+\frac{b^{2/3}x^2}{a^{2/3}}}$$

$$\left((-1)^{2/3}\,\text{EllipticE}\Big[\text{ArcSin}\Big[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\Big],(-1)^{1/3}\Big]-\right.$$

$$\left.i\,\text{EllipticF}\Big[\text{ArcSin}\Big[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\Big],(-1)^{1/3}\Big]\right)$$

Problem 101: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\sqrt{-a+b x^3}}\,dx$$

Optimal (type 4, 497 leaves, 3 steps):

$$\frac{2\sqrt{-a+b x^3}}{b^{1/3}\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)}-\left(3^{1/4}\sqrt{2+\sqrt{3}}\;a^{1/3}\left(a^{1/3}-b^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{1/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\sqrt{-a+b x^3}\right)+$$

$$\left(4\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Big/$$

$$\left(b^{1/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\sqrt{-a+b x^3}\right)$$

Result (type 4, 257 leaves):

$$\left(2\,(-a)^{1/3}\sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}(-b)^{1/3}x\right)}{a}}\;\sqrt{1+\frac{(-b)^{1/3}x\left((-a)^{1/3}+(-b)^{1/3}x\right)}{(-a)^{2/3}}}\right.$$

$$\left(3\,(-1)^{2/3}(-a)^{1/3}b^{1/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$i\left(\left(-3+\sqrt{3}\right)a^{1/3}(-b)^{1/3}-\sqrt{3}\,(-a)^{1/3}b^{1/3}\right)$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(3^{3/4}(-b)^{2/3}\sqrt{-a+b x^3}\right)$$

Problem 102: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\sqrt{-a-b x^3}}\, dx$$

Optimal (type 4, 488 leaves, 3 steps):

$$-\frac{2 \sqrt{-a-b x^3}}{b^{1/3} \left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\left(3^{1/4} \sqrt{2+\sqrt{3}}\, a^{1/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7+4 \sqrt{3}\right]\right)\Big/$$

$$\left(b^{1/3} \sqrt{-\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \sqrt{-a-b x^3}\right)-$$

$$\left(4\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{1/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7+4 \sqrt{3}\right]\right)\Big/$$

$$\left(b^{1/3} \sqrt{-\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \sqrt{-a-b x^3}\right)$$

Result (type 4, 227 leaves):

$$\left(2\,i\,(-a)^{1/3}\sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}b^{1/3}x\right)}{a}}\sqrt{1+\frac{b^{1/3}x\left((-a)^{1/3}+b^{1/3}x\right)}{(-a)^{2/3}}}\right.$$

$$\left(-3\,(-1)^{1/6}(-a)^{1/3}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left(\sqrt{3}\,(-a)^{1/3}+\left(-3+\sqrt{3}\right)a^{1/3}\right)$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(3^{3/4}\,b^{1/3}\sqrt{-a-b\,x^3}\right)$$

Problem 103: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1-\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 241 leaves, 1 step):

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+b\,x^3}}{b\left(1+\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x\right)}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\left(1+\left(\frac{b}{a}\right)^{1/3}x\right)\sqrt{\frac{1-\left(\frac{b}{a}\right)^{1/3}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1+\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x}{1+\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x}\right],\right.\right.$$

$$\left.\left.-7-4\sqrt{3}\right]\right)\Big/\left(\left(\frac{b}{a}\right)^{1/3}\sqrt{\frac{1+\left(\frac{b}{a}\right)^{1/3}x}{\left(1+\sqrt{3}+\left(\frac{b}{a}\right)^{1/3}x\right)^2}}\sqrt{a+b\,x^3}\right)$$

Result (type 4, 243 leaves):

$$\left(2\,i\,a^{1/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3}x}{a^{1/3}}+\frac{(-b)^{2/3}x^2}{a^{2/3}}}\right.$$

$$\left(-3\,(-1)^{1/6}a^{1/3}\left(\frac{b}{a}\right)^{1/3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left(\left(-3+\sqrt{3}\right)(-b)^{1/3}+\sqrt{3}\,a^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(3^{3/4}(-b)^{2/3}\sqrt{a+b\,x^3}\right)$$

Problem 104: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1-\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x}{\sqrt{a-b\,x^3}}\,dx$$

Optimal (type 4, 248 leaves, 1 step):

$$-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-b\,x^3}}{b\left(1+\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x\right)}+$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\left(1-\left(\frac{b}{a}\right)^{1/3}x\right)\sqrt{\frac{1+\left(\frac{b}{a}\right)^{1/3}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1+\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x}{1+\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x}\right],\right.\right.$$

$$\left.\left.-7-4\sqrt{3}\right]\right)\Big/\left(\left(\frac{b}{a}\right)^{1/3}\sqrt{\frac{1-\left(\frac{b}{a}\right)^{1/3}x}{\left(1+\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x\right)^2}}\sqrt{a-b\,x^3}\right)$$

Result (type 4, 232 leaves):

$$\left(2\,a^{1/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+b^{1/3}\,x\right)}{a^{1/3}}}\;\sqrt{1+\frac{b^{1/3}\,x}{a^{1/3}}+\frac{b^{2/3}\,x^2}{a^{2/3}}}\right.$$

$$\left(3\,(-1)^{2/3}\,a^{1/3}\left(\frac{b}{a}\right)^{1/3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+\right.$$

$$i\left(\left(-3+\sqrt{3}\right)b^{1/3}-\sqrt{3}\,a^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\Big/\left(3^{3/4}\,b^{2/3}\sqrt{a-b\,x^3}\right)$$

Problem 105: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1-\sqrt{3}-\left(\frac{b}{a}\right)^{1/3}x}{\sqrt{-a+b\,x^3}}\,dx$$

Optimal (type 4, 549 leaves, 3 steps):

$$\frac{2\left(\frac{b}{a}\right)^{1/3}\sqrt{-a+b x^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)}-\left(3^{1/4}\sqrt{2+\sqrt{3}}\;a^{1/3}\left(\frac{b}{a}\right)^{1/3}\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Big/\left(b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\sqrt{-a+b x^3}\right)-\left(2\sqrt{2-\sqrt{3}}\;\left(\left(1-\sqrt{3}\right)b^{1/3}-\left(1+\sqrt{3}\right)a^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)\left(a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{2/3}+a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}x}{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x}\right],-7+4\sqrt{3}\right]\right)\Big/\left(3^{1/4}b^{2/3}\sqrt{-\frac{a^{1/3}\left(a^{1/3}-b^{1/3}x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}x\right)^2}}\;\sqrt{-a+b x^3}\right)$$

Result (type 4, 267 leaves):

$$\left(2\,(-a)^{1/3}\sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}(-b)^{1/3}x\right)}{a}}\sqrt{1+\frac{(-b)^{1/3}x\left((-a)^{1/3}+(-b)^{1/3}x\right)}{(-a)^{2/3}}}\left(3\,(-1)^{2/3}(-a)^{1/3}\left(\frac{b}{a}\right)^{1/3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+i\left(-3\,(-b)^{1/3}+\sqrt{3}\,(-b)^{1/3}-\sqrt{3}\,(-a)^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(3^{3/4}(-b)^{2/3}\sqrt{-a+b x^3}\right)$$

Problem 106: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1-\sqrt{3}+\left(\frac{b}{a}\right)^{1/3} x}{\sqrt{-a-b x^3}} \, dx$$

Optimal (type 4, 540 leaves, 3 steps):

$$-\frac{2 \left(\frac{b}{a}\right)^{1/3} \sqrt{-a-b x^3}}{b^{2/3} \left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\left(3^{1/4} \sqrt{2+\sqrt{3}}\; a^{1/3} \left(\frac{b}{a}\right)^{1/3} \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$
$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7+4 \sqrt{3}\right]\right) \Big/$$
$$\left(b^{2/3} \sqrt{-\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{-a-b x^3}\right)+$$
$$\left(2 \sqrt{2-\sqrt{3}} \left(\left(1-\sqrt{3}\right) b^{1/3}-\left(1+\sqrt{3}\right) a^{1/3} \left(\frac{b}{a}\right)^{1/3}\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7+4 \sqrt{3}\right]\right) \Big/$$
$$\left(3^{1/4} b^{2/3} \sqrt{-\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{-a-b x^3}\right)$$

Result (type 4, 245 leaves):

$$\left(2\,\mathrm{i}\,(-a)^{1/3}\sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}b^{1/3}x\right)}{a}}\sqrt{1+\frac{b^{1/3}x\left((-a)^{1/3}+b^{1/3}x\right)}{(-a)^{2/3}}}\right.$$

$$\left(-3\,(-1)^{1/6}(-a)^{1/3}\left(\frac{b}{a}\right)^{1/3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,b^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]+\right.$$

$$\left(\left(-3+\sqrt{3}\right)b^{1/3}+\sqrt{3}\,(-a)^{1/3}\left(\frac{b}{a}\right)^{1/3}\right)$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{\mathrm{i}\,b^{1/3}x}{(-a)^{1/3}}}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)\right)\Big/\left(3^{3/4}\,b^{2/3}\sqrt{-a-b\,x^3}\right)$$

Problem 107: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 490 leaves, 3 steps):

$$\frac{2\,d\sqrt{a+b\,x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\,a^{1/3}\,d\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\right.\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b\,x^3}\right)+$$

$$\left(2\sqrt{2+\sqrt{3}}\left(b^{1/3}c-\left(1-\sqrt{3}\right)a^{1/3}d\right)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4}\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b\,x^3}\right)$$

Result (type 4, 221 leaves):

$$-\left(\left(2\,a^{1/3}\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\left((-1)^{2/3}\sqrt{3}\,a^{1/3}\,d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-i\left((-b)^{1/3}\,c+a^{1/3}\,d\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\Big/\left(3^{1/4}\,(-b)^{2/3}\sqrt{a+b\,x^3}\right)\right)$$

Problem 108: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x}{\sqrt{a-b\,x^3}}\,dx$$

Optimal (type 4, 503 leaves, 3 steps):

$$\frac{2\,d\,\sqrt{a-b\,x^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)}-$$

$$\left(3^{1/4}\,\sqrt{2-\sqrt{3}}\;a^{1/3}\,d\,\left(a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\right.\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Bigg/\left(b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\sqrt{a-b\,x^3}\right)-$$

$$\left(2\sqrt{2+\sqrt{3}}\;\left(b^{1/3}\,c+\left(1-\sqrt{3}\right)a^{1/3}\,d\right)\left(a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x}\right],\,-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(3^{1/4}\,b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}-b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}-b^{1/3}\,x\right)^2}}\;\sqrt{a-b\,x^3}\right)$$

Result (type 4, 208 leaves):

$$-\left(\left(2\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+b^{1/3}\,x\right)}{a^{1/3}}}\;\sqrt{1+\frac{b^{1/3}\,x}{a^{1/3}}+\frac{b^{2/3}\,x^2}{a^{2/3}}}\right.\right.$$

$$\left((-1)^{2/3}\sqrt{3}\;a^{2/3}\,d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-i\,a^{1/3}\left(b^{1/3}\,c+a^{1/3}\,d\right)\right.$$

$$\left.\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\Bigg/\left(3^{1/4}\,b^{2/3}\sqrt{a-b\,x^3}\right)\right)$$

Problem 109: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x}{\sqrt{-a+b\,x^3}}\,dx$$

Optimal (type 4, 515 leaves, 3 steps):

$$-\frac{2\,d\,\sqrt{-a+b\,x^3}}{b^{2/3}\,\left(\left(1-\sqrt{3}\right)\,a^{1/3}-b^{1/3}\,x\right)}+\left(3^{1/4}\,\sqrt{2+\sqrt{3}}\,a^{1/3}\,d\,\left(a^{1/3}-b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)\,a^{1/3}-b^{1/3}\,x}{\left(1-\sqrt{3}\right)\,a^{1/3}-b^{1/3}\,x}\right],\,-7+4\,\sqrt{3}\right]\right)\Big/$$

$$\left(b^{2/3}\,\sqrt{-\frac{a^{1/3}\,\left(a^{1/3}-b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-b^{1/3}\,x\right)^2}}\,\sqrt{-a+b\,x^3}\right)-$$

$$\left(2\,\sqrt{2-\sqrt{3}}\,\left(b^{1/3}\,c+\left(1+\sqrt{3}\right)\,a^{1/3}\,d\right)\,\left(a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)\,a^{1/3}-b^{1/3}\,x}{\left(1-\sqrt{3}\right)\,a^{1/3}-b^{1/3}\,x}\right],\,-7+4\,\sqrt{3}\right]\right)\Big/$$

$$\left(3^{1/4}\,b^{2/3}\,\sqrt{-\frac{a^{1/3}\,\left(a^{1/3}-b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)\,a^{1/3}-b^{1/3}\,x\right)^2}}\,\sqrt{-a+b\,x^3}\right)$$

Result (type 4, 236 leaves):

$$-\left(\left(2\,(-a)^{1/3}\,\sqrt{-\frac{(-1)^{5/6}\,\left(a+(-a)^{2/3}\,(-b)^{1/3}\,x\right)}{a}}\right.\right.$$

$$\sqrt{1+\frac{(-b)^{1/3}\,x\,\left((-a)^{1/3}+(-b)^{1/3}\,x\right)}{(-a)^{2/3}}}\,\left((-1)^{2/3}\,\sqrt{3}\,(-a)^{1/3}\,d\right.$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{(-a)^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-i\,\left((-b)^{1/3}\,c+(-a)^{1/3}\,d\right)$$

$$\left.\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{(-a)^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\Big/\left(3^{1/4}\,(-b)^{2/3}\,\sqrt{-a+b\,x^3}\right)\right)$$

Problem 110: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c+d\,x}{\sqrt{-a-b\,x^3}}\,dx$$

Optimal (type 4, 508 leaves, 3 steps):

$$-\frac{2\,d\,\sqrt{-a-b\,x^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+\left(3^{1/4}\,\sqrt{2+\sqrt{3}}\;a^{1/3}\,d\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7+4\,\sqrt{3}\right]\right)\Bigg/\left(b^{2/3}\,\sqrt{-\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{-a-b\,x^3}\right)+\left(2\,\sqrt{2-\sqrt{3}}\;\left(b^{1/3}\,c-\left(1+\sqrt{3}\right)a^{1/3}\,d\right)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7+4\,\sqrt{3}\right]\right)\Bigg/\left(3^{1/4}\,b^{2/3}\,\sqrt{-\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{-a-b\,x^3}\right)$$

Result (type 4, 223 leaves):

$$-\left(\left(2\,(-a)^{1/3}\,\sqrt{-\frac{(-1)^{5/6}\left(a+(-a)^{2/3}\,b^{1/3}\,x\right)}{a}}\;\sqrt{1+\frac{b^{1/3}\,x\left((-a)^{1/3}+b^{1/3}\,x\right)}{(-a)^{2/3}}}\;\left((-1)^{2/3}\,\sqrt{3}\;(-a)^{1/3}\,d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}\,x}{(-a)^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-i\left(b^{1/3}\,c+(-a)^{1/3}\,d\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,b^{1/3}\,x}{(-a)^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)\right)\Bigg/\left(3^{1/4}\,b^{2/3}\,\sqrt{-a-b\,x^3}\right)\right)$$

Problem 111: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c+d\,x}{\sqrt{1+x^3}}\,dx$$

Optimal (type 4, 246 leaves, 3 steps):

$$\frac{2\,d\,\sqrt{1+x^3}}{1+\sqrt{3}+x}-\left(3^{1/4}\,\sqrt{2-\sqrt{3}}\;d\,(1+x)\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\,\sqrt{3}\right]\right)\Big/\left(\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\;\sqrt{1+x^3}\right)+\left(2\,\sqrt{2+\sqrt{3}}\;\left(c-\left(1-\sqrt{3}\right)d\right)\,(1+x)\,\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right],-7-4\,\sqrt{3}\right]\right)\Big/\left(3^{1/4}\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\;\sqrt{1+x^3}\right)$$

Result (type 4, 136 leaves):

$$-\frac{1}{3^{3/4}\,\sqrt{1+x^3}}2\,\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}\;\sqrt{1+(-1)^{1/3}\,x+(-1)^{2/3}\,x^2}\;\left(3\,d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}\,(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]+(-1)^{1/6}\,\sqrt{3}\,\left(-c+(-1)^{2/3}\,d\right)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}\,(1+x)}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 112: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c+d\,x}{\sqrt{1-x^3}}\,dx$$

Optimal (type 4, 271 leaves, 3 steps):

$$\frac{2\,d\,\sqrt{1-x^3}}{1+\sqrt{3}-x}-\left(3^{1/4}\sqrt{2-\sqrt{3}}\,d\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}\right)-\left(2\sqrt{2+\sqrt{3}}\,\left(c+d-\sqrt{3}\,d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(3^{1/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}\right)$$

Result (type 4, 121 leaves):

$$\frac{1}{3^{3/4}\sqrt{1-x^3}}2\,\mathrm{i}\,\sqrt{(-1)^{5/6}(-1+x)}\,\sqrt{1+x+x^2}\left(-3\,(-1)^{1/6}\,d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\mathrm{i}\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]+\sqrt{3}\,(c+d)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\mathrm{i}\,x}}{3^{1/4}}\right],(-1)^{1/3}\right]\right)$$

Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x}{\sqrt{-1+x^3}}\,dx$$

Optimal (type 4, 275 leaves, 3 steps):

$$-\frac{2\,d\,\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\left(3^{1/4}\sqrt{2+\sqrt{3}}\,d\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]\right)\Big/\left(\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}\right)-\left(2\sqrt{2-\sqrt{3}}\,\left(c+d+\sqrt{3}\,d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right],-7+4\sqrt{3}\right]\right)\Big/\left(3^{1/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}\right)$$

Result (type 4, 119 leaves):

$$\frac{1}{3^{3/4}\sqrt{-1+x^3}}2\,\mathrm{i}\,\sqrt{(-1)^{5/6}(-1+x)}\,\sqrt{1+x+x^2}\left(-3\,(-1)^{1/6}\,d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\mathrm{i}\,x}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+\sqrt{3}\,(c+d)\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\mathrm{i}\,x}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 114: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x}{\sqrt{-1-x^3}}\,dx$$

Optimal (type 4, 261 leaves, 3 steps):

$$-\frac{2\,d\,\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\left(3^{1/4}\sqrt{2+\sqrt{3}}\;d\,(1+x)\,\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],\,-7+4\sqrt{3}\right]\right)\Bigg/\left(\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\;\sqrt{-1-x^3}\right)+\left(2\sqrt{2-\sqrt{3}}\;\left(c-\left(1+\sqrt{3}\right)d\right)(1+x)\,\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right],\,-7+4\sqrt{3}\right]\right)\Bigg/\left(3^{1/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\;\sqrt{-1-x^3}\right)$$

Result (type 4, 152 leaves):

$$\frac{1}{3^{3/4}\sqrt{-1-x^3}}2\,(-1)^{1/6}\sqrt{-(-1)^{5/6}+\mathrm{i}\,x}\;\sqrt{1-(-1)^{2/3}x-(-1)^{1/3}x^2}\left(3\,(-1)^{1/6}\,d\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+\sqrt{3}\left((-1)^{2/3}c-d\right)\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{1/6}\left((-1)^{2/3}+x\right)}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 124: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c+d\,x}{1+x^4}\,dx$$

Optimal (type 3, 98 leaves, 13 steps):

$$\frac{1}{2}\,d\,\text{ArcTan}\left[x^2\right]-\frac{c\,\text{ArcTan}\left[1-\sqrt{2}\,x\right]}{2\,\sqrt{2}}+\frac{c\,\text{ArcTan}\left[1+\sqrt{2}\,x\right]}{2\,\sqrt{2}}-\frac{c\,\text{Log}\left[1-\sqrt{2}\,x+x^2\right]}{4\,\sqrt{2}}+\frac{c\,\text{Log}\left[1+\sqrt{2}\,x+x^2\right]}{4\,\sqrt{2}}$$

Result (type 3, 99 leaves):

$$\frac{1}{4}\left(-\left((-1)^{1/4}\,c+i\,d\right)\text{Log}\left[(-1)^{1/4}-x\right]+\left(-(-1)^{3/4}\,c+i\,d\right)\text{Log}\left[(-1)^{3/4}-x\right]+\left((-1)^{1/4}\,c-i\,d\right)\text{Log}\left[(-1)^{1/4}+x\right]+\left((-1)^{3/4}\,c+i\,d\right)\text{Log}\left[(-1)^{3/4}+x\right]\right)$$

Problem 161: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{b\,x+d\,x^3}{2+3\,x^4}\,dx$$

Optimal (type 3, 36 leaves, 5 steps):

$$\frac{b\,\text{ArcTan}\left[\sqrt{\frac{3}{2}}\,x^2\right]}{2\,\sqrt{6}}+\frac{1}{12}\,d\,\text{Log}\left[2+3\,x^4\right]$$

Result (type 3, 65 leaves):

$$\frac{1}{24}\left(i\,\sqrt{6}\,b+2\,d\right)\text{Log}\left[\sqrt{6}-3\,i\,x^2\right]+\frac{1}{24}\left(-i\,\sqrt{6}\,b+2\,d\right)\text{Log}\left[\sqrt{6}+3\,i\,x^2\right]$$

Problem 210: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c+d\,x}{\sqrt{a+b\,x^4}}\,dx$$

Optimal (type 4, 121 leaves, 6 steps):

$$\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{2\,\sqrt{b}}+\frac{c\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\,b^{1/4}\,\sqrt{a+b\,x^4}}$$

Result (type 4, 107 leaves):

$$\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{2\,\sqrt{b}}-\frac{i\,c\,\sqrt{1+\frac{b\,x^4}{a}}\,\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\,x\right],\,-1\right]}{\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\,\sqrt{a+b\,x^4}}$$

Problem 211: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x}{\sqrt{a-b\,x^4}}\,dx$$

Optimal (type 4, 87 leaves, 7 steps):

$$\frac{d\,\text{ArcTan}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a-b\,x^4}}\right]}{2\,\sqrt{b}}+\frac{a^{1/4}\,c\,\sqrt{1-\frac{b\,x^4}{a}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,-1\right]}{b^{1/4}\,\sqrt{a-b\,x^4}}$$

Result (type 4, 106 leaves):

$$\frac{d\,\text{ArcTan}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a-b\,x^4}}\right]}{2\,\sqrt{b}}-\frac{i\,c\,\sqrt{1-\frac{b\,x^4}{a}}\,\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\,\,x\right],\,-1\right]}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\,\,\sqrt{a-b\,x^4}}$$

Problem 212: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x}{\sqrt{-a+b\,x^4}}\,dx$$

Optimal (type 4, 89 leaves, 7 steps):

$$\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{-a+b\,x^4}}\right]}{2\,\sqrt{b}}+\frac{a^{1/4}\,c\,\sqrt{1-\frac{b\,x^4}{a}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,-1\right]}{b^{1/4}\,\sqrt{-a+b\,x^4}}$$

Result (type 4, 108 leaves):

$$\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{-a+b\,x^4}}\right]}{2\,\sqrt{b}}-\frac{i\,c\,\sqrt{1-\frac{b\,x^4}{a}}\,\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\,\,x\right],\,-1\right]}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\,\,\sqrt{-a+b\,x^4}}$$

Problem 213: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x}{\sqrt{-a-b\,x^4}}\,dx$$

Optimal (type 4, 127 leaves, 6 steps):

$$\frac{d\,\text{ArcTan}\left[\frac{\sqrt{b}\,x^2}{\sqrt{-a-b\,x^4}}\right]}{2\sqrt{b}}+\frac{c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\,b^{1/4}\sqrt{-a-b\,x^4}}$$

Result (type 4, 113 leaves):

$$\frac{d\,\text{ArcTan}\left[\frac{\sqrt{b}\,x^2}{\sqrt{-a-b\,x^4}}\right]}{2\sqrt{b}}-\frac{i\,c\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{-a-b\,x^4}}$$

Problem 214: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x+e\,x^2}{\sqrt{a+b\,x^4}}\,dx$$

Optimal (type 4, 257 leaves, 8 steps):

$$\frac{e\,x\sqrt{a+b\,x^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{2\sqrt{b}}-$$

$$\frac{a^{1/4}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{b^{3/4}\sqrt{a+b\,x^4}}+\frac{1}{2\,b^{3/4}\sqrt{a+b\,x^4}}$$

$$a^{1/4}\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 201 leaves):

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,d\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]+\right.$$

$$2\sqrt{a}\,e\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-2\left(i\sqrt{b}\,c+\sqrt{a}\,e\right)$$

$$\left.\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)\Big/\left(2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b}\sqrt{a+b\,x^4}\right)$$

Problem 220: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x+e\,x^2+f\,x^3+g\,x^4+h\,x^5+i\,x^6}{\sqrt{a+b\,x^4}}\,dx$$

Optimal (type 4, 385 leaves, 12 steps):

$$\frac{f\sqrt{a+b x^4}}{2 b}+\frac{g x\sqrt{a+b x^4}}{3 b}+\frac{h x^2\sqrt{a+b x^4}}{4 b}+\frac{i x^3\sqrt{a+b x^4}}{5 b}+$$
$$\frac{(5 b e-3 a i) x\sqrt{a+b x^4}}{5 b^{3/2}\left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{(2 b d-a h)\,\text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{4 b^{3/2}}-\frac{1}{5 b^{7/4}\sqrt{a+b x^4}}$$
$$a^{1/4}(5 b e-3 a i)\left(\sqrt{a}+\sqrt{b} x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]+$$
$$\frac{1}{30 b^{7/4}\sqrt{a+b x^4}}a^{1/4}\left(15 b e+\frac{5\sqrt{b}\,(3 b c-a g)}{\sqrt{a}}-9 a i\right)$$
$$\left(\sqrt{a}+\sqrt{b} x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 275 leaves):

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{b}\,(a+b x^4)\,(30 f+x(20 g+3 x(5 h+4 i x)))+\right.\right.$$
$$\left.\left.15(2 b d-a h)\sqrt{a+b x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]\right)-\right.$$
$$12\sqrt{a}\,(-5 b e+3 a i)\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+$$
$$4\left(-15 i b^{3/2} c-15\sqrt{a}\,b e+5 i a\sqrt{b}\,g+9 a^{3/2} i\right)\sqrt{1+\frac{b x^4}{a}}$$
$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)\Big/\left(60\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,b^{3/2}\sqrt{a+b x^4}\right)$$

Problem 221: Result is not expressed in closed-form.

$$\int\frac{1+x}{1+x^5}\,dx$$

Optimal (type 3, 109 leaves, 3 steps):

$$-\frac{1}{5}(-1)^{1/5}\left(1+(-1)^{1/5}\right)\text{Log}\left[(-1)^{1/5}-x\right]+\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4/5}\right)\text{Log}\left[-(-1)^{4/5}-x\right]+$$
$$\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\text{Log}\left[(-1)^{2/5}+x\right]-\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\text{Log}\left[-(-1)^{3/5}+x\right]$$

Result (type 7, 51 leaves):

$$\text{RootSum}\left[1-\#1+\#1^2-\#1^3+\#1^4\ \&,\ \frac{\text{Log}[x-\#1]}{-1+2\,\#1-3\,\#1^2+4\,\#1^3}\ \&\right]$$

Problem 222: Result is not expressed in closed-form.

$$\int \frac{1-x}{1-x^5}\,dx$$

Optimal (type 3, 109 leaves, 3 steps):

$$-\frac{1}{5}(-1)^{2/5}\left(1-(-1)^{2/5}\right)\text{Log}\left[(-1)^{2/5}-x\right]+\frac{1}{5}(-1)^{3/5}\left(1+(-1)^{3/5}\right)\text{Log}\left[-(-1)^{3/5}-x\right]+\frac{1}{5}(-1)^{1/5}\left(1+(-1)^{1/5}\right)\text{Log}\left[(-1)^{1/5}+x\right]-\frac{1}{5}(-1)^{4/5}\left(1-(-1)^{4/5}\right)\text{Log}\left[-(-1)^{4/5}+x\right]$$

Result (type 7, 47 leaves):

$$\text{RootSum}\left[1+\#1+\#1^2+\#1^3+\#1^4\ \&,\ \frac{\text{Log}[x-\#1]}{1+2\,\#1+3\,\#1^2+4\,\#1^3}\ \&\right]$$

Problem 369: Result more than twice size of optimal antiderivative.

$$\int \frac{x\left(-2\left(\frac{a}{b}\right)^{1/3}C+C\,x\right)}{a+b\,x^3}\,dx$$

Optimal (type 3, 50 leaves, 4 steps):

$$\frac{2\,C\,\text{ArcTan}\left[\frac{1-\frac{2x}{\left(\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b}+\frac{C\,\text{Log}\left[\left(\frac{a}{b}\right)^{1/3}+x\right]}{b}$$

Result (type 3, 146 leaves):

$$\frac{1}{3\,a^{1/3}\,b}C\left(2\sqrt{3}\left(\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,b^{1/3}\,x}{a^{1/3}}}{\sqrt{3}}\right]+2\left(\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{Log}\left[a^{1/3}+b^{1/3}\,x\right]-\left(\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{Log}\left[a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2\right]+a^{1/3}\,\text{Log}\left[a+b\,x^3\right]\right)$$

Problem 370: Result more than twice size of optimal antiderivative.

$$\int \frac{x\left(-2\left(-\frac{a}{b}\right)^{1/3}C+C\,x\right)}{a-b\,x^3}\,dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$-\frac{2\,C\,\text{ArcTan}\left[\frac{1-\frac{2x}{\left(-\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b}-\frac{C\,\text{Log}\left[\left(-\frac{a}{b}\right)^{1/3}+x\right]}{b}$$

Result (type 3, 149 leaves):

$$-\frac{1}{3\,a^{1/3}\,b}C\left(-2\sqrt{3}\left(-\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{a^{1/3}}}{\sqrt{3}}\right]-2\left(-\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{Log}\left[a^{1/3}-b^{1/3}\,x\right]+\left(-\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{Log}\left[a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2\right]+a^{1/3}\,\text{Log}\left[a-b\,x^3\right]\right)$$

Problem 371: Result more than twice size of optimal antiderivative.

$$\int\frac{x\left(2\left(-\frac{a}{b}\right)^{1/3}C+C\,x\right)}{a+b\,x^3}\,dx$$

Optimal (type 3, 54 leaves, 4 steps):

$$\frac{2\,C\,\text{ArcTan}\left[\frac{1+\frac{2x}{\left(-\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b}+\frac{C\,\text{Log}\left[\left(-\frac{a}{b}\right)^{1/3}-x\right]}{b}$$

Result (type 3, 148 leaves):

$$\frac{1}{3\,a^{1/3}\,b}C\left(-2\sqrt{3}\left(-\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{ArcTan}\left[\frac{1-\frac{2\,b^{1/3}\,x}{a^{1/3}}}{\sqrt{3}}\right]-2\left(-\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{Log}\left[a^{1/3}+b^{1/3}\,x\right]+\left(-\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{Log}\left[a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2\right]+a^{1/3}\,\text{Log}\left[a+b\,x^3\right]\right)$$

Problem 372: Result more than twice size of optimal antiderivative.

$$\int\frac{x\left(2\left(\frac{a}{b}\right)^{1/3}C+C\,x\right)}{a-b\,x^3}\,dx$$

Optimal (type 3, 53 leaves, 4 steps):

$$-\frac{2\,C\,\text{ArcTan}\left[\frac{1+\frac{2x}{\left(\frac{a}{b}\right)^{1/3}}}{\sqrt{3}}\right]}{\sqrt{3}\,b}-\frac{C\,\text{Log}\left[\left(\frac{a}{b}\right)^{1/3}-x\right]}{b}$$

Result (type 3, 147 leaves):

$$-\frac{1}{3\,a^{1/3}\,b}C\left(2\sqrt{3}\,\left(\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{ArcTan}\left[\frac{1+\frac{2\,b^{1/3}\,x}{a^{1/3}}}{\sqrt{3}}\right]+2\,\left(\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{Log}\left[a^{1/3}-b^{1/3}\,x\right]-\left(\frac{a}{b}\right)^{1/3}b^{1/3}\,\text{Log}\left[a^{2/3}+a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2\right]+a^{1/3}\,\text{Log}\left[a-b\,x^3\right]\right)$$

Problem 430: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^3\,\left(c+d\,x+e\,x^2\right)}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 583 leaves, 10 steps):

$$-\frac{4\,a\,e\,\sqrt{a+b\,x^3}}{9\,b^2}+\frac{2\,c\,x\,\sqrt{a+b\,x^3}}{5\,b}+\frac{2\,d\,x^2\,\sqrt{a+b\,x^3}}{7\,b}+\frac{2\,e\,x^3\,\sqrt{a+b\,x^3}}{9\,b}-\frac{8\,a\,d\,\sqrt{a+b\,x^3}}{7\,b^{5/3}\,\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+\left(4\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{4/3}\,d\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Bigg/\left(7\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-\left(4\,\sqrt{2+\sqrt{3}}\,a\,\left(7\,b^{1/3}\,c-10\,\left(1-\sqrt{3}\right)a^{1/3}\,d\right)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],-7-4\sqrt{3}\right]\right)\Bigg/\left(35\times 3^{1/4}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 329 leaves):

$$\frac{1}{315\,(-b)^{8/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\,(a+b\,x^3)\,\left(-70\,a\,e+b\,x\,\left(63\,c+5\,x\,(9\,d+7\,e\,x)\right)\right)+\right.$$
$$360\,(-1)^{2/3}\,3^{1/4}\,a^{5/3}\,b\,d\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$
$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-$$
$$12\,i\,3^{3/4}\,a^{4/3}\,b\,\left(7\,(-b)^{1/3}\,c+10\,a^{1/3}\,d\right)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$
$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 431: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\,(c+d\,x+e\,x^2)}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 560 leaves, 8 steps):

$$\frac{2\,c\,\sqrt{a+b\,x^3}}{3\,b}+\frac{2\,d\,x\,\sqrt{a+b\,x^3}}{5\,b}+\frac{2\,e\,x^2\,\sqrt{a+b\,x^3}}{7\,b}-\frac{8\,a\,e\,\sqrt{a+b\,x^3}}{7\,b^{5/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+\left(4\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\;a^{4/3}\,e\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/\left(7\,b^{5/3}\,\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)-\left(4\,\sqrt{2+\sqrt{3}}\;a\left(7\,b^{1/3}\,d-10\left(1-\sqrt{3}\right)a^{1/3}\,e\right)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Big/\left(35\times 3^{1/4}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 319 leaves) :

$$-\frac{1}{105\,(-b)^{5/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\,(a+b\,x^3)\,(35\,c+3\,x\,(7\,d+5\,e\,x))+120\,(-1)^{2/3}\,3^{1/4}\,a^{5/3}\,e\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\right.$$

$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-$$

$$4\,i\,3^{3/4}\,a^{4/3}\,\left(7\,(-b)^{1/3}\,d+10\,a^{1/3}\,e\right)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 432: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\,(c+d\,x+e\,x^2)}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 537 leaves, 6 steps) :

$$\frac{2\,d\,\sqrt{a+b\,x^3}}{3\,b}+\frac{2\,e\,x\,\sqrt{a+b\,x^3}}{5\,b}+\frac{2\,c\,\sqrt{a+b\,x^3}}{b^{2/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-$$

$$\left(3^{1/4}\,\sqrt{2-\sqrt{3}}\,\,a^{1/3}\,c\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Big/$$

$$\left(b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)-$$

$$\left(2\,\sqrt{2+\sqrt{3}}\,\,a^{1/3}\,\left(5\,\left(1-\sqrt{3}\right)\,b^{2/3}\,c+2\,a^{2/3}\,e\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Big/$$

$$\left(5\times 3^{1/4}\,b^{4/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 314 leaves):

$$\frac{1}{15\,(-b)^{5/3}\,\sqrt{a+b\,x^3}}\left(-2\,(-b)^{2/3}\,(5\,d+3\,e\,x)\,\left(a+b\,x^3\right)+30\,(-1)^{2/3}\,3^{1/4}\,a^{2/3}\,b\,c\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\right.$$

$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+$$

$$2\,i\,3^{3/4}\,a^{2/3}\,\left(-5\,b\,c+2\,a^{2/3}\,(-b)^{1/3}\,e\right)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 433: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c+d\,x+e\,x^2}{\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 509 leaves, 5 steps):

$$\frac{2 e \sqrt{a+b x^3}}{3 b}+\frac{2 d \sqrt{a+b x^3}}{b^{2/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-$$

$$\left(3^{1/4} \sqrt{2-\sqrt{3}}\ a^{1/3} d \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$

$$\left(2 \sqrt{2+\sqrt{3}} \left(b^{1/3} c-\left(1-\sqrt{3}\right) a^{1/3} d\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right], -7-4 \sqrt{3}\right]\right) \Big/$$

$$\left(3^{1/4} b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 305 leaves):

$$-\left(\left(2 (-b)^{2/3} e \left(a+b x^3\right)-6 (-1)^{2/3} 3^{1/4} a^{2/3} b d \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\right.\right.$$

$$\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]+$$

$$2 i\, 3^{3/4} a^{1/3} b \left((-b)^{1/3} c+a^{1/3} d\right) \sqrt{\frac{(-1)^{5/6} \left(-a^{1/3}+(-b)^{1/3} x\right)}{a^{1/3}}} \sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], (-1)^{1/3}\right]\right) \Big/ \left(3 (-b)^{5/3} \sqrt{a+b x^3}\right)\right)$$

Problem 434: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c + d\,x + e\,x^2}{x\,\sqrt{a + b\,x^3}}\,dx$$

Optimal (type 4, 518 leaves, 7 steps):

$$\frac{2\,e\,\sqrt{a+b\,x^3}}{b^{2/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)} - \frac{2\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{3\,\sqrt{a}} - \left(3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{1/3}\,e\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/\left(b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right) + \left(2\,\sqrt{2+\sqrt{3}}\,\left(b^{1/3}\,d-\left(1-\sqrt{3}\right)\,a^{1/3}\,e\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/\left(3^{1/4}\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 493 leaves):

$$-\frac{2\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{3\,\sqrt{a}}-$$

$$\left(2\,d\,\left(\left(-1\right)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{\left(-1\right)^{1/3}\,a^{1/3}-\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\text{EllipticF}\left[\right.\right.$$

$$\left.\left.\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\right],\,\left(-1\right)^{1/3}\right]\right)\Big/\left(b^{1/3}\,\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(2\,\sqrt{2}\,a^{1/3}\,e\,\left(\left(-1\right)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{\left(-1\right)^{1/3}\,a^{1/3}-\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{i\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}\right.$$

$$\left(\left(-1+\left(-1\right)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-1\right)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{\left(-1\right)^{1/3}}{-1+\left(-1\right)^{1/3}}\right]+\text{EllipticF}\left[\right.\right.$$

$$\left.\left.\left.\text{ArcSin}\left[\frac{\sqrt{\left(-1\right)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{\left(-1\right)^{1/3}}{-1+\left(-1\right)^{1/3}}\right]\right)\right)\Big/\left(b^{2/3}\,\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right)$$

Problem 435: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x+e\,x^2}{x^2\,\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 547 leaves, 8 steps):

$$-\frac{c\sqrt{a+b x^3}}{a x}+\frac{b^{1/3} c \sqrt{a+b x^3}}{a\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{2 d \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3\sqrt{a}}-$$

$$\left(3^{1/4}\sqrt{2-\sqrt{3}}\, b^{1/3} c \left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\bigg/$$

$$\left(2 a^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)-$$

$$\left(\sqrt{2+\sqrt{3}}\left(\left(1-\sqrt{3}\right) b^{2/3} c-2 a^{2/3} e\right)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\bigg/$$

$$\left(3^{1/4} a^{2/3} b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 513 leaves):

$$-\frac{c\sqrt{a+b\,x^3}}{a\,x}-\frac{2\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{3\sqrt{a}}-$$

$$\left(2\,e\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\;\text{EllipticF}\left[\right.\right.$$

$$\left.\left.\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],\,(-1)^{1/3}\right]\right)\Big/\left(b^{1/3}\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{a+b\,x^3}\right)-$$

$$\left(\sqrt{2}\,b^{1/3}\,c\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{\frac{i\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}\right.$$

$$\left(\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\text{EllipticF}\left[\right.\right.$$

$$\left.\left.\left.\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\Big/\left(a^{2/3}\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{a+b\,x^3}\right)$$

Problem 436: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x+e\,x^2}{x^3\sqrt{a+b\,x^3}}\,dx$$

Optimal (type 4, 569 leaves, 9 steps):

$$-\frac{c\sqrt{a+b x^3}}{2 a x^2}-\frac{d\sqrt{a+b x^3}}{a x}+\frac{b^{1/3} d\sqrt{a+b x^3}}{a\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{2 e \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3\sqrt{a}}-\left(3^{1/4}\sqrt{2-\sqrt{3}}\ b^{1/3} d\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Bigg/\left(2 a^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)-\left(\sqrt{2+\sqrt{3}}\ b^{1/3}\left(b^{1/3} c+2\left(1-\sqrt{3}\right) a^{1/3} d\right)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Bigg/\left(2\times 3^{1/4} a\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 525 leaves) :

$$-\frac{\left(c+2\,d\,x\right)\sqrt{a+b\,x^3}}{2\,a\,x^2}-\frac{2\,e\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{3\,\sqrt{a}}+$$

$$\left(b^{2/3}\,c\,\left(\left(-1\right)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{\left(-1\right)^{1/3}\,a^{1/3}-\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\right],\,\left(-1\right)^{1/3}\right]\right)\Bigg/\left(2\,a\,\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(\sqrt{2}\,b^{1/3}\,d\,\left(\left(-1\right)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{\left(-1\right)^{1/3}\,a^{1/3}-\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{i\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}\right.$$

$$\left(\left(-1+\left(-1\right)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-1\right)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{\left(-1\right)^{1/3}}{-1+\left(-1\right)^{1/3}}\right]+\text{EllipticF}\left[\right.\right.$$

$$\left.\left.\left.\text{ArcSin}\left[\frac{\sqrt{\left(-1\right)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{\left(-1\right)^{1/3}}{-1+\left(-1\right)^{1/3}}\right]\right)\right)\Bigg/\left(a^{2/3}\,\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right)$$

Problem 437: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^5\,\left(c+d\,x+e\,x^2\right)}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 594 leaves, 8 steps):

$$\frac{2 x \left(a d+a e x-b c x^2\right)}{3 b^2 \sqrt{a+b x^3}}+\frac{4 c \sqrt{a+b x^3}}{3 b^2}+\frac{2 d x \sqrt{a+b x^3}}{5 b^2}+\frac{2 e x^2 \sqrt{a+b x^3}}{7 b^2}-\frac{80 a e \sqrt{a+b x^3}}{21 b^{8/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\left(40 \sqrt{2-\sqrt{3}}\; a^{4/3} e \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(7\times 3^{3/4} b^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-\left(16 \sqrt{2+\sqrt{3}}\; a \left(14 b^{1/3} d-25 \left(1-\sqrt{3}\right) a^{1/3} e\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/\left(105\times 3^{1/4} b^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 334 leaves):

$$\frac{1}{315\,(-b)^{8/3}\sqrt{a+b\,x^3}}\left(6\,(-b)^{2/3}\left(a\left(70\,c+56\,d\,x+50\,e\,x^2\right)+b\,x^3\left(35\,c+3\,x\left(7\,d+5\,e\,x\right)\right)\right)+\right.$$

$$1200\,(-1)^{2/3}\,3^{1/4}\,a^{5/3}\,e\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-$$

$$16\,i\,3^{3/4}\,a^{4/3}\left(14\,(-b)^{1/3}\,d+25\,a^{1/3}\,e\right)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 438: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4\left(c+d\,x+e\,x^2\right)}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 574 leaves, 7 steps):

$$\frac{2 x \left(a e-b c x-b d x^2\right)}{3 b^2 \sqrt{a+b x^3}}+\frac{4 d \sqrt{a+b x^3}}{3 b^2}+\frac{2 e x \sqrt{a+b x^3}}{5 b^2}+$$
$$\frac{8 c \sqrt{a+b x^3}}{3 b^{5/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\left(4 \sqrt{2-\sqrt{3}}\; a^{1/3} c \left(a^{1/3}+b^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(3^{3/4} b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \sqrt{a+b x^3}\right)-$$
$$\left(8 \sqrt{2+\sqrt{3}}\; a^{1/3} \left(5 \left(1-\sqrt{3}\right) b^{2/3} c+4 a^{2/3} e\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(15\times 3^{1/4} b^{7/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \sqrt{a+b x^3}\right)$$

Result (type 4, 330 leaves) :

$$\frac{1}{45 \left(-b\right)^{8/3} \sqrt{a+b x^3}} \left(6 \left(-b\right)^{2/3} \left(2 a \left(5 d+4 e x\right)+b x^2 \left(-5 c+5 d x+3 e x^2\right)\right) - \right.$$

$$120 \left(-1\right)^{2/3} 3^{1/4} a^{2/3} b c \sqrt{\left(-1\right)^{5/6} \left(-1+\frac{\left(-b\right)^{1/3} x}{a^{1/3}}\right)} \sqrt{1+\frac{\left(-b\right)^{1/3} x}{a^{1/3}}+\frac{\left(-b\right)^{2/3} x^2}{a^{2/3}}}$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{i \left(-b\right)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \left(-1\right)^{1/3}\right] -$$

$$8 i\, 3^{3/4} a^{2/3} \left(-5 b c+4 a^{2/3} \left(-b\right)^{1/3} e\right) \sqrt{\frac{\left(-1\right)^{5/6} \left(-a^{1/3}+\left(-b\right)^{1/3} x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{\left(-b\right)^{1/3} x}{a^{1/3}}+\frac{\left(-b\right)^{2/3} x^2}{a^{2/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{i \left(-b\right)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \left(-1\right)^{1/3}\right]\right)$$

Problem 439: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 \left(c+d x+e x^2\right)}{\left(a+b x^3\right)^{3/2}} \, dx$$

Optimal (type 4, 542 leaves, 6 steps):

$$-\frac{2\,x\,\left(c+d\,x+e\,x^2\right)}{3\,b\,\sqrt{a+b\,x^3}}+\frac{4\,e\,\sqrt{a+b\,x^3}}{3\,b^2}+\frac{8\,d\,\sqrt{a+b\,x^3}}{3\,b^{5/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-$$

$$\left(4\,\sqrt{2-\sqrt{3}}\;a^{1/3}\,d\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Bigg/$$

$$\left(3^{3/4}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+$$

$$\left(4\,\sqrt{2+\sqrt{3}}\;\left(b^{1/3}\,c-2\,\left(1-\sqrt{3}\right)\,a^{1/3}\,d\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Bigg/$$

$$\left(3\times3^{1/4}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 319 leaves) :

$$\frac{1}{9\,(-b)^{8/3}\sqrt{a+b\,x^3}}\left(6\,(-b)^{2/3}\left(2\,a\,e+b\,x\,\left(-c-d\,x+e\,x^2\right)\right)-24\,(-1)^{2/3}\,3^{1/4}\,a^{2/3}\,b\,d\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\right.$$

$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+$$

$$4\,i\,3^{3/4}\,a^{1/3}\,b\,\left((-b)^{1/3}\,c+2\,a^{1/3}\,d\right)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 440: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\left(c+d\,x+e\,x^2\right)}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 522 leaves, 4 steps):

$$-\frac{2\left(c+d\,x+e\,x^2\right)}{3\,b\,\sqrt{a+b\,x^3}}+\frac{8\,e\,\sqrt{a+b\,x^3}}{3\,b^{5/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}-$$

$$\left(4\,\sqrt{2-\sqrt{3}}\;a^{1/3}\,e\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(3^{3/4}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)+$$

$$\left(4\,\sqrt{2+\sqrt{3}}\;\left(b^{1/3}\,d-2\left(1-\sqrt{3}\right)a^{1/3}\,e\right)\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(3\times 3^{1/4}\,b^{5/3}\,\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 305 leaves):

$$\frac{1}{9\left(-b\right)^{5/3}\sqrt{a+b\,x^3}}\left(6\left(-b\right)^{2/3}\left(c+x\left(d+e\,x\right)\right)+24\left(-1\right)^{2/3}\,3^{1/4}\,a^{2/3}\,e\,\sqrt{\left(-1\right)^{5/6}\left(-1+\frac{\left(-b\right)^{1/3}x}{a^{1/3}}\right)}\right.$$

$$\sqrt{1+\frac{\left(-b\right)^{1/3}x}{a^{1/3}}+\frac{\left(-b\right)^{2/3}x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{i\,\left(-b\right)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\left(-1\right)^{1/3}\right]-$$

$$4\,i\,3^{3/4}\,a^{1/3}\left(\left(-b\right)^{1/3}d+2\,a^{1/3}\,e\right)\sqrt{\frac{\left(-1\right)^{5/6}\left(-a^{1/3}+\left(-b\right)^{1/3}x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{\left(-b\right)^{1/3}x}{a^{1/3}}+\frac{\left(-b\right)^{2/3}x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\left(-1\right)^{5/6}-\frac{i\,\left(-b\right)^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\left(-1\right)^{1/3}\right]\right)$$

Problem 441: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x\left(c+d x+e x^2\right)}{\left(a+b x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 561 leaves, 6 steps):

$$-\frac{2 x\left(a e-b c x-b d x^2\right)}{3 a b \sqrt{a+b x^3}}-\frac{2 d \sqrt{a+b x^3}}{3 a b}-\frac{2 c \sqrt{a+b x^3}}{3 a b^{2/3}\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\left(\sqrt{2-\sqrt{3}}\; c\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Bigg/\left(3^{3/4} a^{2/3} b^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)+\left(2\sqrt{2+\sqrt{3}}\left(b^{2/3}\left(c-\sqrt{3}\, c\right)+2 a^{2/3} e\right)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Bigg/\left(3\times 3^{1/4} a^{2/3} b^{4/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 317 leaves):

$$-\frac{1}{9\,a\,(-b)^{5/3}\sqrt{a+b\,x^3}}\left(6\,(-b)^{2/3}\,\left(b\,c\,x^2-a\,(d+e\,x)\right)+6\,(-1)^{2/3}\,3^{1/4}\,a^{2/3}\,b\,c\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\right.$$

$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+$$

$$2\,i\,3^{3/4}\,a^{2/3}\,\left(-b\,c+2\,a^{2/3}\,(-b)^{1/3}\,e\right)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 442: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x+e\,x^2}{\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 532 leaves, 4 steps):

$$-\frac{2 d \sqrt{a+b x^3}}{3 a b^{2/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{2 \left(a e-b x \left(c+d x\right)\right)}{3 a b \sqrt{a+b x^3}}+$$

$$\left(\sqrt{2-\sqrt{3}}\; d \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(3^{3/4} a^{2/3} b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$

$$\left(2 \sqrt{2+\sqrt{3}} \left(b^{1/3} c+\left(1-\sqrt{3}\right) a^{1/3} d\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(3\times 3^{1/4} a b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 314 leaves):

$$-\frac{1}{9\, a\, (-b)^{5/3} \sqrt{a+b\, x^3}}\left(6\, (-b)^{2/3} \left(-a\, e+b\, x\, (c+d\, x)\right)+6\, (-1)^{2/3}\, 3^{1/4}\, a^{2/3}\, b\, d\, \sqrt{(-1)^{5/6} \left(-1+\frac{(-b)^{1/3} x}{a^{1/3}}\right)}\right.$$

$$\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\, (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\ (-1)^{1/3}\right]+$$

$$2\, i\, 3^{3/4}\, a^{1/3}\, b \left((-b)^{1/3} c-a^{1/3} d\right) \sqrt{\frac{(-1)^{5/6} \left(-a^{1/3}+(-b)^{1/3} x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3} x}{a^{1/3}}+\frac{(-b)^{2/3} x^2}{a^{2/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\, (-b)^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\ (-1)^{1/3}\right]\right)$$

Problem 443: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c+d\, x+e\, x^2}{x \left(a+b\, x^3\right)^{3/2}}\, dx$$

Optimal (type 4, 579 leaves, 10 steps):

$$\frac{2 x \left(a d + a e x - b c x^2\right)}{3 a^2 \sqrt{a + b x^3}} + \frac{2 c \sqrt{a + b x^3}}{3 a^2} - \frac{2 e \sqrt{a + b x^3}}{3 a b^{2/3} \left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)} -$$

$$\frac{2 c \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3 a^{3/2}} + \left(\sqrt{2 - \sqrt{3}}\; e \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Bigg/$$

$$\left(3^{3/4} a^{2/3} b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right) +$$

$$\left(2 \sqrt{2 + \sqrt{3}} \left(b^{1/3} d + \left(1 - \sqrt{3}\right) a^{1/3} e\right) \left(a^{1/3} + b^{1/3} x\right) \sqrt{\frac{a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1 - \sqrt{3}\right) a^{1/3} + b^{1/3} x}{\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x}\right], -7 - 4 \sqrt{3}\right]\right) \Bigg/$$

$$\left(3 \times 3^{1/4} a b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3} + b^{1/3} x\right)}{\left(\left(1 + \sqrt{3}\right) a^{1/3} + b^{1/3} x\right)^2}} \sqrt{a + b x^3}\right)$$

Result (type 4, 518 leaves):

$$\frac{1}{3a}2\left(\frac{c+x\,(d+e\,x)}{\sqrt{a+b\,x^3}}-\frac{c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{\sqrt{a}}-\right.$$

$$\left(d\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\text{EllipticF}\Big[\right.$$

$$\left.\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],\,(-1)^{1/3}\Big]\right)\Bigg/\left(b^{1/3}\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{a+b\,x^3}\right)+$$

$$\left(\sqrt{2}\,a^{1/3}\,e\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{i\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}\right.$$

$$\left(\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\right.$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\Bigg/$$

$$\left.\left(b^{2/3}\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{a+b\,x^3}\right)\right)$$

Problem 444: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x+e\,x^2}{x^2\,\left(a+b\,x^3\right)^{3/2}}\,dx$$

Optimal (type 4, 607 leaves, 11 steps):

$$\frac{2x\left(a e-b c x-b d x^{2}\right)}{3 a^{2}\sqrt{a+b x^{3}}}+\frac{2 d\sqrt{a+b x^{3}}}{3 a^{2}}-\frac{c\sqrt{a+b x^{3}}}{a^{2} x}+\frac{5 b^{1/3} c\sqrt{a+b x^{3}}}{3 a^{2}\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{2 d \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^{3}}}{\sqrt{a}}\right]}{3 a^{3/2}}-\left(5\sqrt{2-\sqrt{3}}\, b^{1/3} c\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^{2}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^{2}}}\, \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(2\times 3^{3/4} a^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^{2}}}\sqrt{a+b x^{3}}\right)-\left(\sqrt{2+\sqrt{3}}\left(5\left(1-\sqrt{3}\right) b^{2/3} c-2 a^{2/3} e\right)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^{2}}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^{2}}}\, \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(3\times 3^{1/4} a^{5/3} b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^{2}}}\sqrt{a+b x^{3}}\right)$$

Result (type 4, 542 leaves) :

$$\frac{-3\,a\,c-5\,b\,c\,x^3+2\,a\,x\,(d+e\,x)}{3\,a^2\,x\,\sqrt{a+b\,x^3}}-\frac{1}{6\,a^2}\Bigg(4\,\sqrt{a}\,d\,\text{ArcTanh}\Big[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\Big]+\Bigg(4\,a\,e\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}$$

$$\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\;\text{EllipticF}\Big[\text{ArcSin}\Big[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\Big],\,(-1)^{1/3}\Big]\Bigg)\Bigg/$$

$$\left(b^{1/3}\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right)+$$

$$\Bigg(10\,\sqrt{2}\,a^{1/3}\,b^{1/3}\,c\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{i\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\Bigg(\left(-1+(-1)^{2/3}\right)\,\text{EllipticE}\Big[\text{ArcSin}\Big[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\Big],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\Big]+\text{EllipticF}\Big[$$

$$\text{ArcSin}\Big[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\Big],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\Big]\Bigg)\Bigg)\Bigg/\left(\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right)\Bigg)$$

Problem 445: Result unnecessarily involves imaginary or complex numbers.

$$\int x^3\,\sqrt{a+b\,x^3}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)\,dx$$

Optimal (type 4, 733 leaves, 13 steps) :

$$-\frac{4 a^2 e \sqrt{a+b x^3}}{45 b^2}+\frac{6 a (17 b c-8 a f) x \sqrt{a+b x^3}}{935 b^2}+\frac{6 a (19 b d-10 a g) x^2 \sqrt{a+b x^3}}{1729 b^2}+$$

$$\frac{2 a e x^3 \sqrt{a+b x^3}}{45 b}+\frac{6 a f x^4 \sqrt{a+b x^3}}{187 b}+\frac{6 a g x^5 \sqrt{a+b x^3}}{247 b}-\frac{24 a^2 (19 b d-10 a g) \sqrt{a+b x^3}}{1729 b^{8/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+$$

$$\frac{1}{692835} 2 x^3 \sqrt{a+b x^3} \left(62985 c x+53295 d x^2+46189 e x^3+40755 f x^4+36465 g x^5\right)+$$

$$\left(12\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{7/3} (19 b d-10 a g) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(1729 b^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-$$

$$\left(4\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a^2 \left(1729 b^{1/3} (17 b c-8 a f)-1870 \left(1-\sqrt{3}\right) a^{1/3} (19 b d-10 a g)\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(1616615 b^{8/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 433 leaves) :

$$\frac{1}{14549535\,(-b)^{8/3}\sqrt{a+b\,x^3}}\Bigg(2\,(-b)^{2/3}\,(a+b\,x^3)\,\Big(-2\,a^2\,(323323\,e+27\,x\,(6916\,f+4675\,g\,x))+21\,b^2\,x^4$$
$$(62985\,c+11\,x\,(4845\,d+13\,x\,(323\,e+285\,f\,x+255\,g\,x^2)))+$$
$$a\,b\,x\,(793611\,c+x\,(479655\,d+7\,x\,(46189\,e+135\,x\,(247\,f+187\,g\,x))))\Big)+$$
$$201960\,(-1)^{2/3}\,3^{1/4}\,a^{8/3}\,(19\,b\,d-10\,a\,g)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$
$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-$$
$$36\,i\,3^{3/4}\,a^{7/3}\left(323\,b\left(91\,(-b)^{1/3}\,c+110\,a^{1/3}\,d\right)-4\left(3458\,a\,(-b)^{1/3}\,f+4675\,a^{4/3}\,g\right)\right)$$
$$\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\;\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$
$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\Bigg)$$

Problem 446: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2\,\sqrt{a+b\,x^3}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)\,dx$$

Optimal (type 4, 681 leaves, 11 steps):

$$\frac{2a(5bc-2af)\sqrt{a+bx^3}}{45b^2}+\frac{6a(17bd-8ag)x\sqrt{a+bx^3}}{935b^2}+\frac{6aex^2\sqrt{a+bx^3}}{91b}+$$

$$\frac{2afx^3\sqrt{a+bx^3}}{45b}+\frac{6agx^4\sqrt{a+bx^3}}{187b}-\frac{24a^2e\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}+$$

$$\frac{2x^2\sqrt{a+bx^3}\left(12155cx+9945dx^2+8415ex^3+7293fx^4+6435gx^5\right)}{109395}+$$

$$\left(12\times 3^{1/4}\sqrt{2-\sqrt{3}}\,a^{7/3}e\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(91b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)-$$

$$\left(4\times 3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(1547bd-1870\left(1-\sqrt{3}\right)a^{1/3}b^{2/3}e-728ag\right)\left(a^{1/3}+b^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(85085b^{7/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)$$

Result (type 4, 399 leaves) :

$$\frac{1}{765765\,(-b)^{8/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\,(a+b\,x^3)\right.$$

$$\left(-182\,a^2\,(187\,f+108\,g\,x)+7\,b^2\,x^3\,(12155\,c+9945\,d\,x+33\,x^2\,(255\,e+13\,x\,(17\,f+15\,g\,x)))+\right.$$
$$\left.a\,b\,(85085\,c+x\,(41769\,d+x\,(25245\,e+17017\,f\,x+12285\,g\,x^2)))\right)+$$

$$201960\,(-1)^{2/3}\,3^{1/4}\,a^{8/3}\,b\,e\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-$$

$$36\,i\,3^{3/4}\,a^{7/3}\,\left(17\,b\,\left(91\,(-b)^{1/3}\,d+110\,a^{1/3}\,e\right)-728\,a\,(-b)^{1/3}\,g\right)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 447: Result unnecessarily involves imaginary or complex numbers.

$$\int x\,\sqrt{a+b\,x^3}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)\,dx$$

Optimal (type 4, 667 leaves, 9 steps):

$$\frac{2\,a\,(5\,b\,d-2\,a\,g)\,\sqrt{a+b\,x^3}}{45\,b^2}+\frac{6\,a\,e\,x\,\sqrt{a+b\,x^3}}{55\,b}+$$

$$\frac{6\,a\,f\,x^2\,\sqrt{a+b\,x^3}}{91\,b}+\frac{2\,a\,g\,x^3\,\sqrt{a+b\,x^3}}{45\,b}+\frac{6\,a\,(13\,b\,c-4\,a\,f)\,\sqrt{a+b\,x^3}}{91\,b^{5/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}+$$

$$\frac{2\,x\,\sqrt{a+b\,x^3}\,\left(6435\,c\,x+5005\,d\,x^2+4095\,e\,x^3+3465\,f\,x^4+3003\,g\,x^5\right)}{45\,045}-$$

$$\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^{4/3}\,(13\,b\,c-4\,a\,f)\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(91\,b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)-$$

$$\left(2\times 3^{3/4}\sqrt{2+\sqrt{3}}\;a^{4/3}\left(182\,a^{2/3}\,b^{1/3}\,e+55\left(1-\sqrt{3}\right)(13\,b\,c-4\,a\,f)\right)\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\right]\right)\Bigg/$$

$$\left(5005\,b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\;\sqrt{a+b\,x^3}\right)$$

Result (type 4, 390 leaves) :

$$\frac{1}{45045\,(-b)^{8/3}\,\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\,\left(a+b\,x^3\right)\,\left(-2002\,a^2\,g+b^2\,x^2\,\left(6435\,c+7\,x\,\left(715\,d+585\,e\,x+495\,f\,x^2+429\,g\,x^3\right)\right)+a\,b\,\left(5005\,d+x\,\left(2457\,e+11\,x\,\left(135\,f+91\,g\,x\right)\right)\right)\right)-2970\,(-1)^{2/3}\,3^{1/4}\,a^{5/3}\,b\,\left(13\,b\,c-4\,a\,f\right)\,\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-18\,i\,3^{3/4}\,a^{5/3}\,b\,\left(-715\,b\,c+182\,a^{2/3}\,(-b)^{1/3}\,e+220\,a\,f\right)\,\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 448: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{a+b\,x^3}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)\,dx$$

Optimal (type 4, 639 leaves, 8 steps):

$$\frac{2\,a\,e\,\sqrt{a+b\,x^3}}{9\,b}+\frac{6\,a\,f\,x\,\sqrt{a+b\,x^3}}{55\,b}+\frac{6\,a\,g\,x^2\,\sqrt{a+b\,x^3}}{91\,b}+\frac{6\,a\,\left(13\,b\,d-4\,a\,g\right)\,\sqrt{a+b\,x^3}}{91\,b^{5/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+$$

$$\frac{2\,\sqrt{a+b\,x^3}\,\left(9009\,c\,x+6435\,d\,x^2+5005\,e\,x^3+4095\,f\,x^4+3465\,g\,x^5\right)}{45\,045}-$$

$$\left(3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,\,a^{4/3}\,\left(13\,b\,d-4\,a\,g\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Big/$$

$$\left(91\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)+$$

$$\left(2\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,\,a\,\left(91\,b^{1/3}\,\left(11\,b\,c-2\,a\,f\right)-55\,\left(1-\sqrt{3}\right)\,a^{1/3}\,\left(13\,b\,d-4\,a\,g\right)\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Big/$$

$$\left(5005\,b^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 393 leaves):

$$-\frac{1}{45045\,(-b)^{5/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\,(a+b\,x^3)\right.$$

$$\left(a\,(5005\,e+27\,x\,(91\,f+55\,g\,x))+b\,x\,(9009\,c+5\,x\,(1287\,d+7\,x\,(143\,e+117\,f\,x+99\,g\,x^2)))\right)-$$

$$2970\,(-1)^{2/3}\,3^{1/4}\,a^{5/3}\,(13\,b\,d-4\,a\,g)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+$$

$$18\,i\,3^{3/4}\,a^{4/3}\left(143\,b\left(7\,(-b)^{1/3}\,c+5\,a^{1/3}\,d\right)-2\,a\left(91\,(-b)^{1/3}\,f+110\,a^{1/3}\,g\right)\right)$$

$$\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\;\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 449: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{a+b\,x^3}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)}{x}\,dx$$

Optimal (type 4, 620 leaves, 11 steps):

$$\frac{2\,a\,f\,\sqrt{a+b\,x^3}}{9\,b}+\frac{6\,a\,g\,x\,\sqrt{a+b\,x^3}}{55\,b}+\frac{6\,a\,e\,\sqrt{a+b\,x^3}}{7\,b^{2/3}\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+$$

$$\frac{2\,\sqrt{a+b\,x^3}\,\left(1155\,c\,x+693\,d\,x^2+495\,e\,x^3+385\,f\,x^4+315\,g\,x^5\right)}{3465\,x}-$$

$$\frac{2}{3}\,\sqrt{a}\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-\left(3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{4/3}\,e\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\bigg/$$

$$\left(7\,b^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+$$

$$\left(2\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,a\,\left(77\,b\,d-55\,\left(1-\sqrt{3}\right)\,a^{1/3}\,b^{2/3}\,e-14\,a\,g\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\bigg/$$

$$\left(385\,b^{4/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 714 leaves) :

$$\frac{1}{3465\,b}2\sqrt{a+b\,x^3}\,\left(1155\,b\,c+7\,a\,\left(55\,f+27\,g\,x\right)+b\,x\,\left(693\,d+5\,x\,\left(99\,e+7\,x\,\left(11\,f+9\,g\,x\right)\right)\right)\right)-$$

$$\frac{1}{1155\,b^{4/3}\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}}$$

$$2\,\sqrt{a}\,\left(385\,b^{4/3}\,c\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]+\right.$$

$$693\,\sqrt{a}\,b\,d\,\left(\left(-1\right)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{\left(-1\right)^{1/3}\,\left(a^{1/3}-\left(-1\right)^{1/3}\,b^{1/3}\,x\right)}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\right],\,\left(-1\right)^{1/3}\right]-$$

$$126\,a^{3/2}\,g\,\left(\left(-1\right)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{\left(-1\right)^{1/3}\,\left(a^{1/3}-\left(-1\right)^{1/3}\,b^{1/3}\,x\right)}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\right],\,\left(-1\right)^{1/3}\right]-$$

$$495\,\sqrt{2}\,a^{5/6}\,b^{2/3}\,e\,\left(\left(-1\right)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{\left(-1\right)^{1/3}\,\left(a^{1/3}-\left(-1\right)^{1/3}\,b^{1/3}\,x\right)}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{i\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left(-\left(-1+\left(-1\right)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-1\right)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{\left(-1\right)^{1/3}}{-1+\left(-1\right)^{1/3}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-1\right)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{\left(-1\right)^{1/3}}{-1+\left(-1\right)^{1/3}}\right]\right)\right)$$

Problem 450: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{a+b\,x^3}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)}{x^2}\,dx$$

Optimal (type 4, 638 leaves, 11 steps):

$$\frac{2 a g \sqrt{a+b x^3}}{9 b}-\frac{3 c \sqrt{a+b x^3}}{x}+\frac{3 (7 b c+2 a f) \sqrt{a+b x^3}}{7 b^{2/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+$$
$$\frac{2 \sqrt{a+b x^3} \left(315 c x+105 d x^2+63 e x^3+45 f x^4+35 g x^5\right)}{315 x^2}-$$
$$\frac{2}{3} \sqrt{a}\, d \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-\left(3\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{1/3} (7 b c+2 a f) \left(a^{1/3}+b^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(14 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$
$$\left(3^{3/4} \sqrt{2+\sqrt{3}}\, a^{1/3} \left(14 a^{2/3} b^{1/3} e-5 \left(1-\sqrt{3}\right) (7 b c+2 a f)\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(35 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 810 leaves):

$$\frac{1}{315 b x \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}}$$
$$\left(\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \left(a+b x^3\right) \left(-315 b c+70 a g x+2 b x (105 d+x (63 e+5 x (9 f+7 g x)))\right)-\right.$$
$$210 \sqrt{a}\, b d x \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-$$

$$378\, a\, b^{2/3}\, e\, x\, \left(\left(-1\right)^{1/3} a^{1/3} - b^{1/3} x\right) \sqrt{\frac{a^{1/3} + b^{1/3} x}{\left(1 + \left(-1\right)^{1/3}\right) a^{1/3}}}$$

$$\sqrt{\frac{\left(-1\right)^{1/3} \left(a^{1/3} - \left(-1\right)^{1/3} b^{1/3} x\right)}{\left(1 + \left(-1\right)^{1/3}\right) a^{1/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3} + \left(-1\right)^{2/3} b^{1/3} x}{\left(1 + \left(-1\right)^{1/3}\right) a^{1/3}}}\right],\ \left(-1\right)^{1/3}\right] +$$

$$945 \sqrt{2}\, a^{1/3}\, b^{4/3}\, c\, x\, \left(\left(-1\right)^{1/3} a^{1/3} - b^{1/3} x\right) \sqrt{\frac{\left(-1\right)^{1/3} \left(a^{1/3} - \left(-1\right)^{1/3} b^{1/3} x\right)}{\left(1 + \left(-1\right)^{1/3}\right) a^{1/3}}}$$

$$\sqrt{\frac{i \left(1 + \frac{b^{1/3} x}{a^{1/3}}\right)}{3\, i + \sqrt{3}}} \left(-\left(-1 + \left(-1\right)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-1\right)^{1/6} - \frac{i\, b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{\left(-1\right)^{1/3}}{-1 + \left(-1\right)^{1/3}}\right] - \right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-1\right)^{1/6} - \frac{i\, b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{\left(-1\right)^{1/3}}{-1 + \left(-1\right)^{1/3}}\right]\right) +$$

$$270 \sqrt{2}\, a^{4/3}\, b^{1/3}\, f\, x\, \left(\left(-1\right)^{1/3} a^{1/3} - b^{1/3} x\right) \sqrt{\frac{\left(-1\right)^{1/3} \left(a^{1/3} - \left(-1\right)^{1/3} b^{1/3} x\right)}{\left(1 + \left(-1\right)^{1/3}\right) a^{1/3}}}$$

$$\sqrt{\frac{i \left(1 + \frac{b^{1/3} x}{a^{1/3}}\right)}{3\, i + \sqrt{3}}} \left(-\left(-1 + \left(-1\right)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-1\right)^{1/6} - \frac{i\, b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{\left(-1\right)^{1/3}}{-1 + \left(-1\right)^{1/3}}\right] - \right.$$

$$\left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-1\right)^{1/6} - \frac{i\, b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{\left(-1\right)^{1/3}}{-1 + \left(-1\right)^{1/3}}\right]\right)\right)$$

Problem 451: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + b x^3}\, \left(c + d x + e x^2 + f x^3 + g x^4\right)}{x^3}\, dx$$

Optimal (type 4, 640 leaves, 10 steps):

$$\frac{3 c \sqrt{a+b x^3}}{2 x^2}-\frac{3 d \sqrt{a+b x^3}}{x}+\frac{3 (7 b d+2 a g) \sqrt{a+b x^3}}{7 b^{2/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-$$

$$\frac{2 \sqrt{a+b x^3} \left(105 c x-105 d x^2-35 e x^3-21 f x^4-15 g x^5\right)}{105 x^3}-$$

$$\frac{2}{3} \sqrt{a}\; e \,\text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-\left(3\times 3^{1/4} \sqrt{2-\sqrt{3}}\; a^{1/3} (7 b d+2 a g) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(14 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \sqrt{a+b x^3}\right)+$$

$$\left(3^{3/4} \sqrt{2+\sqrt{3}} \left(7 b^{1/3} (5 b c+4 a f)-10 \left(1-\sqrt{3}\right) a^{1/3} (7 b d+2 a g)\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(70 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \sqrt{a+b x^3}\right)$$

Result (type 4, 962 leaves):

$$\frac{1}{210 b^{2/3} x^2 \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\; \sqrt{a+b x^3}}$$

$$\left(b^{2/3} \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\; \left(a+b x^3\right) \left(-105 c+2 x \left(-105 d+70 e x+42 f x^2+30 g x^3\right)\right)-\right.$$

$$140 \sqrt{a}\; b^{2/3} e x^2 \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\; \sqrt{a+b x^3}\; \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-$$

$$315 b^{4/3} c x^2 \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}$$

$$\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],(-1)^{1/3}\right]-$$

$$252\,a\,b^{1/3}f\,x^2\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{1/3}+b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\ \sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],(-1)^{1/3}\right]+$$

$$630\sqrt{2}\,a^{1/3}\,b\,d\,x^2\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\ \sqrt{\frac{i\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)+$$

$$180\sqrt{2}\,a^{4/3}\,g\,x^2\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\ \sqrt{\frac{i\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

Problem 452: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^3}\,\left(c+d x+e x^2+f x^3+g x^4\right)}{x^4}\,dx$$

Optimal (type 4, 637 leaves, 11 steps):

$$\frac{c\sqrt{a+b x^3}}{3 x^3}+\frac{3 d\sqrt{a+b x^3}}{2 x^2}-\frac{3 e\sqrt{a+b x^3}}{x}+\frac{3 b^{1/3} e\sqrt{a+b x^3}}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}-$$

$$\frac{2\sqrt{a+b x^3}\,\left(5 c x+15 d x^2-15 e x^3-5 f x^4-3 g x^5\right)}{15 x^4}-\frac{\left(b c+2 a f\right)\text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3\sqrt{a}}-$$

$$\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\;a^{1/3} b^{1/3} e\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\text{EllipticE}\left[\right.\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(2\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\sqrt{a+b x^3}\right)+$$

$$\left(3^{3/4}\sqrt{2+\sqrt{3}}\;\left(5 b d-10\left(1-\sqrt{3}\right) a^{1/3} b^{2/3} e+4 a g\right)\left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(10\, b^{1/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\;\sqrt{a+b x^3}\right)$$

Result (type 4, 769 leaves):

$$\sqrt{a+b x^3}\left(\frac{2 f}{3}-\frac{10 c+3 x\left(5 d+10 e x-4 g x^3\right)}{30 x^3}\right)-$$

$$\frac{b c \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3 \sqrt{a}}-\frac{2}{3} \sqrt{a}\ f \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-$$

$$\left(3 b^{2/3} d\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right.$$

$$\left.\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]\right) \Big/$$

$$\left(2 \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}\right)-\left(6 a g\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right.$$

$$\left.\sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]\right) \Big/$$

$$\left(5 b^{1/3} \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}\right)-$$

$$\left(3 \sqrt{2}\ a^{1/3} b^{1/3} e\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{i\left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}\right.$$

$$\left(\left(-1+(-1)^{2/3}\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\operatorname{EllipticF}\left[\right.\right.$$

$$\left.\left.\left.\operatorname{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right) \Big/ \left(\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}\right)$$

Problem 453: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^3}\left(c+d x+e x^2+f x^3+g x^4\right)}{x^5} \, dx$$

Optimal (type 4, 694 leaves, 12 steps):

$$\frac{3c\sqrt{a+bx^3}}{20x^4}+\frac{d\sqrt{a+bx^3}}{3x^3}+\frac{3e\sqrt{a+bx^3}}{2x^2}-\frac{3\left(bc+8af\right)\sqrt{a+bx^3}}{8ax}+$$
$$\frac{3b^{1/3}\left(bc+8af\right)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}-\frac{2\sqrt{a+bx^3}\left(3cx+5dx^2+15ex^3-15fx^4-5gx^5\right)}{15x^5}-$$
$$\frac{\left(bd+2ag\right)\text{ArcTanh}\left[\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right]}{3\sqrt{a}}-\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\,b^{1/3}\left(bc+8af\right)\left(a^{1/3}+b^{1/3}x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(16a^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)+$$
$$\left(3^{3/4}\sqrt{2+\sqrt{3}}\,b^{1/3}\left(4a^{2/3}b^{1/3}e-\left(1-\sqrt{3}\right)\left(bc+8af\right)\right)\left(a^{1/3}+b^{1/3}x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$
$$\left(8a^{2/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+bx^3}\right)$$

Result (type 4, 855 leaves):

$$\frac{\sqrt{a+bx^3}\left(-6ac-9bcx^3-4ax\left(2d+x\left(3e+6fx-4gx^2\right)\right)\right)}{24ax^4}-$$
$$\frac{1}{24a\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{a+bx^3}}\left(8\sqrt{a}\,bd\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{a+bx^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right]+\right.$$
$$16a^{3/2}g\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{a+bx^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right]+$$
$$36ab^{2/3}e\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{1/3}+b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],(-1)^{1/3}\right]-$$

$$9\sqrt{2}\,a^{1/3}b^{4/3}c\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{\frac{i\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)-$$

$$72\sqrt{2}\,a^{4/3}b^{1/3}f\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{\frac{i\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

Problem 454: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{a+b\,x^3}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)}{x^6}\,dx$$

Optimal (type 4, 652 leaves, 10 steps):

$$-\frac{1}{60}\left(\frac{12\,c}{x^5}+\frac{15\,d}{x^4}+\frac{20\,e}{x^3}+\frac{30\,f}{x^2}+\frac{60\,g}{x}\right)\sqrt{a+b\,x^3}-$$

$$\frac{3\,b\,c\,\sqrt{a+b\,x^3}}{20\,a\,x^2}-\frac{3\,b\,d\,\sqrt{a+b\,x^3}}{8\,a\,x}+\frac{3\,b^{1/3}\,\left(b\,d+8\,a\,g\right)\,\sqrt{a+b\,x^3}}{8\,a\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-$$

$$\frac{b\,e\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{3\,\sqrt{a}}-\left(3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,\,b^{1/3}\,\left(b\,d+8\,a\,g\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Bigg/$$

$$\left(16\,a^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)-$$

$$\left(3^{3/4}\,\sqrt{2+\sqrt{3}}\,\,b^{1/3}\,\left(2\,b^{1/3}\,\left(b\,c-10\,a\,f\right)+5\,\left(1-\sqrt{3}\right)\,a^{1/3}\,\left(b\,d+8\,a\,g\right)\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Bigg/$$

$$\left(40\,a\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 934 leaves):

$$-\frac{1}{120\,a\,x^5}\sqrt{a+b\,x^3}\,\,\left(24\,a\,c+9\,b\,x^3\,\left(2\,c+5\,d\,x\right)+10\,a\,x\,\left(3\,d+4\,e\,x+6\,x^2\,\left(f+2\,g\,x\right)\right)\right)-$$

$$\frac{1}{120\,a\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\,\sqrt{a+b\,x^3}}$$

$$b^{1/3}\left(40\,\sqrt{a}\,\,b^{2/3}\,e\,\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\,\sqrt{a+b\,x^3}\,\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-\right.$$

$$18\,b^{4/3}\,c\,\left(\left(-1\right)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\,\sqrt{\frac{\left(-1\right)^{1/3}\,\left(a^{1/3}-\left(-1\right)^{1/3}\,b^{1/3}\,x\right)}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],\ (-1)^{1/3}\right]+$$

$$180\, a\, b^{1/3} f\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],\ (-1)^{1/3}\right]-$$

$$45\sqrt{2}\ a^{1/3}\, b\, d\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{\frac{i\left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3\, i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\, b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\ \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\, b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\ \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)-$$

$$360\sqrt{2}\ a^{4/3} g\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3} b^{1/3} x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{\frac{i\left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3\, i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\, b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\ \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\, b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\ \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

Problem 455: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^3}\ \left(c+d x+e x^2+f x^3+g x^4\right)}{x^7}\, dx$$

Optimal (type 4, 659 leaves, 11 steps):

$$-\frac{1}{60}\left(\frac{10\,c}{x^6}+\frac{12\,d}{x^5}+\frac{15\,e}{x^4}+\frac{20\,f}{x^3}+\frac{30\,g}{x^2}\right)\sqrt{a+b\,x^3}-\frac{b\,c\,\sqrt{a+b\,x^3}}{12\,a\,x^3}-\frac{3\,b\,d\,\sqrt{a+b\,x^3}}{20\,a\,x^2}-\frac{3\,b\,e\,\sqrt{a+b\,x^3}}{8\,a\,x}+\frac{3\,b^{4/3}\,e\,\sqrt{a+b\,x^3}}{8\,a\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+\frac{b\,\left(b\,c-4\,a\,f\right)\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{12\,a^{3/2}}-\left(3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,b^{4/3}\,e\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Big/\left(16\,a^{2/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)-\left(3^{3/4}\,\sqrt{2+\sqrt{3}}\,b^{2/3}\,\left(2\,b\,d+5\,\left(1-\sqrt{3}\right)\,a^{1/3}\,b^{2/3}\,e-20\,a\,g\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Big/\left(40\,a\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 800 leaves):

$$-\frac{1}{120\,a\,x^6}\sqrt{a+b\,x^3}\,\left(b\,x^3\,\left(10\,c+9\,x\,\left(2\,d+5\,e\,x\right)\right)+a\,\left(20\,c+2\,x\,\left(12\,d+5\,x\,\left(3\,e+4\,f\,x+6\,g\,x^2\right)\right)\right)\right)+$$

$$\frac{1}{80\,a}\,b\,\left(\frac{20\,b\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{3\,\sqrt{a}}-\frac{80}{3}\,\sqrt{a}\,f\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]+\right.$$

$$\left(12\,b^{2/3}\,d\,\left(\left(-1\right)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{\left(-1\right)^{1/3}\,a^{1/3}-\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\right],\,\left(-1\right)^{1/3}\right]\right)\Big/$$

$$\left(\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right)-\left(120\,a\,g\,\left(\left(-1\right)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\right.$$

$$\left.\sqrt{\frac{\left(-1\right)^{1/3}\,a^{1/3}-\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\right],\,\left(-1\right)^{1/3}\right]\right)\Big/$$

$$\left(b^{1/3}\,\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right)-$$

$$\left(30\,\sqrt{2}\,a^{1/3}\,b^{1/3}\,e\,\left(\left(-1\right)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{\left(-1\right)^{1/3}\,a^{1/3}-\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{\mathrm{i}\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,\mathrm{i}+\sqrt{3}}}\right.$$

$$\left(\left(-1+\left(-1\right)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-1\right)^{1/6}-\frac{\mathrm{i}\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{\left(-1\right)^{1/3}}{-1+\left(-1\right)^{1/3}}\right]+\text{EllipticF}\left[\right.\right.$$

$$\left.\left.\left.\text{ArcSin}\left[\frac{\sqrt{\left(-1\right)^{1/6}-\frac{\mathrm{i}\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{\left(-1\right)^{1/3}}{-1+\left(-1\right)^{1/3}}\right]\right)\right)\Big/\left(\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3}\,b^{1/3}\,x}{\left(1+\left(-1\right)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right)\right)$$

Problem 456: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^3}\,\left(c+d x+e x^2+f x^3+g x^4\right)}{x^8}\,dx$$

Optimal (type 4, 711 leaves, 12 steps):

$$-\frac{1}{420}\left(\frac{60 c}{x^7}+\frac{70 d}{x^6}+\frac{84 e}{x^5}+\frac{105 f}{x^4}+\frac{140 g}{x^3}\right)\sqrt{a+b x^3}-\frac{3 b c \sqrt{a+b x^3}}{56 a x^4}-\frac{b d \sqrt{a+b x^3}}{12 a x^3}-\frac{3 b e \sqrt{a+b x^3}}{20 a x^2}+\frac{3 b\,(5 b c-14 a f)\sqrt{a+b x^3}}{112 a^2 x}-\frac{3 b^{4/3}(5 b c-14 a f)\sqrt{a+b x^3}}{112 a^2\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3} x\right)}+\frac{b\,(b d-4 a g)\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{12 a^{3/2}}+$$

$$\left(3\times 3^{1/4}\sqrt{2-\sqrt{3}}\,b^{4/3}(5 b c-14 a f)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3} x\right)^2}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(224 a^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)-$$

$$\left(3^{3/4}\sqrt{2+\sqrt{3}}\,b^{4/3}\left(28 a^{2/3} b^{1/3} e-5\left(1-\sqrt{3}\right)(5 b c-14 a f)\right)\left(a^{1/3}+b^{1/3} x\right)\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3} x\right)^2}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3} x}\right],-7-4\sqrt{3}\right]\right)\Big/\left(560 a^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3} x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 892 leaves):

$$\frac{1}{1680 a^2 x^7}\sqrt{a+b x^3}\left(225 b^2 c x^6-2 a b x^3\left(45 c+7 x\left(10 d+9 x\left(2 e+5 f x\right)\right)\right)-4 a^2\left(60 c+7 x\left(10 d+x\left(12 e+5 x\left(3 f+4 g x\right)\right)\right)\right)\right)+\frac{1}{1680 a^2\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{a+b x^3}}$$

$$b\left(140\sqrt{a}\,b\,d\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-\right.$$

$$560\,a^{3/2}\,g\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]+$$

$$252\,a\,b^{2/3}\,e\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{1/3}+b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],\,(-1)^{1/3}\right]-$$

$$225\sqrt{2}\,a^{1/3}\,b^{4/3}\,c\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\sqrt{\frac{\mathbb{i}\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3\,\mathbb{i}+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{\mathbb{i}\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{\mathbb{i}\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)+$$

$$630\sqrt{2}\,a^{4/3}\,b^{1/3}\,f\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}$$

$$\sqrt{\frac{\mathbb{i}\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3\,\mathbb{i}+\sqrt{3}}}\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{\mathbb{i}\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{\mathbb{i}\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

Problem 457: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b\,x^3}\,\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)}{x^9}\,dx$$

Optimal (type 4, 743 leaves, 13 steps):

$$-\frac{1}{840}\left(\frac{105\,c}{x^8}+\frac{120\,d}{x^7}+\frac{140\,e}{x^6}+\frac{168\,f}{x^5}+\frac{210\,g}{x^4}\right)\sqrt{a+b\,x^3}-\frac{3\,b\,c\,\sqrt{a+b\,x^3}}{80\,a\,x^5}-\frac{3\,b\,d\,\sqrt{a+b\,x^3}}{56\,a\,x^4}-\frac{b\,e\,\sqrt{a+b\,x^3}}{12\,a\,x^3}+\frac{3\,b\,\left(7\,b\,c-16\,a\,f\right)\,\sqrt{a+b\,x^3}}{320\,a^2\,x^2}+\frac{3\,b\,\left(5\,b\,d-14\,a\,g\right)\,\sqrt{a+b\,x^3}}{112\,a^2\,x}-\frac{3\,b^{4/3}\,\left(5\,b\,d-14\,a\,g\right)\,\sqrt{a+b\,x^3}}{112\,a^2\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}+\frac{b^2\,e\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{12\,a^{3/2}}+\left(3\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,b^{4/3}\,\left(5\,b\,d-14\,a\,g\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Big/\left(224\,a^{5/3}\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+\left(3^{3/4}\,\sqrt{2+\sqrt{3}}\,b^{4/3}\,\left(7\,b^{1/3}\,\left(7\,b\,c-16\,a\,f\right)+20\,\left(1-\sqrt{3}\right)\,a^{1/3}\,\left(5\,b\,d-14\,a\,g\right)\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],-7-4\,\sqrt{3}\right]\right)\Big/\left(2240\,a^2\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 979 leaves):

$$\frac{1}{6720\,a^2\,x^8}\sqrt{a+b\,x^3}\,\left(9\,b^2\,x^6\,\left(49\,c+100\,d\,x\right)-4\,a\,b\,x^3\,\left(63\,c+2\,x\,\left(45\,d+7\,x\,\left(10\,e+9\,x\,\left(2\,f+5\,g\,x\right)\right)\right)\right)-8\,a^2\,\left(105\,c+2\,x\,\left(60\,d+7\,x\,\left(10\,e+3\,x\,\left(4\,f+5\,g\,x\right)\right)\right)\right)\right)+\frac{1}{6720\,a^2\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}}$$

$$b^{4/3}\left(560\sqrt{a}\,b^{2/3}\,e\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-\right.$$

$$441\,b^{4/3}\,c\,\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{1/3}+b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],\,(-1)^{1/3}\right]+$$

$$1008\,a\,b^{1/3}\,f\,\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{1/3}+b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],\,(-1)^{1/3}\right]-$$

$$900\sqrt{2}\,a^{1/3}\,b\,d\,\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{i\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)+$$

$$2520\sqrt{2}\,a^{4/3}\,g\,\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{i\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\right.$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\Bigg)\Bigg)$$

Problem 458: Result unnecessarily involves imaginary or complex numbers.

$$\int x^3\left(a+b x^3\right)^{3/2}\left(c+d x+e x^2+f x^3+g x^4\right)dx$$

Optimal (type 4, 791 leaves, 14 steps):

$$-\frac{4 a^3 e\sqrt{a+b x^3}}{105 b^2}+\frac{54 a^2(23 b c-8 a f)x\sqrt{a+b x^3}}{21505 b^2}+\frac{54 a^2(5 b d-2 a g)x^2\sqrt{a+b x^3}}{8645 b^2}+\frac{2 a^2 e x^3\sqrt{a+b x^3}}{105 b}+\frac{54 a^2 f x^4\sqrt{a+b x^3}}{4301 b}+\frac{54 a^2 g x^5\sqrt{a+b x^3}}{6175 b}-\frac{216 a^3(5 b d-2 a g)\sqrt{a+b x^3}}{8645 b^{8/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}+\frac{1}{3900225}$$

$$2 x^3\left(a+b x^3\right)^{3/2}\left(229425 c x+205275 d x^2+185725 e x^3+169575 f x^4+156009 g x^5\right)+\frac{1}{185910725}$$

$$2 a x^3\sqrt{a+b x^3}\left(8947575 c x+6774075 d x^2+5311735 e x^3+4279275 f x^4+3522519 g x^5\right)+$$

$$\left(108\times 3^{1/4}\sqrt{2-\sqrt{3}}\,a^{10/3}(5 b d-2 a g)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(8645 b^{8/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)-$$

$$\left(36\times 3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(1729 b^{1/3}(23 b c-8 a f)-8602\left(1-\sqrt{3}\right)a^{1/3}(5 b d-2 a g)\right)\left(a^{1/3}+b^{1/3}x\right)\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(37182145 b^{8/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 466 leaves):

$$\frac{1}{557\,732\,175\,(-b)^{8/3}\sqrt{a+b\,x^3}}\Bigg(2\,(-b)^{2/3}\,(a+b\,x^3)\,\Big(-10\,a^3\,\big(1\,062\,347\,e+81\,x\,(6916\,f+4301\,g\,x)\big)+a^2\,b\,x\,\big(16\,105\,635\,c+x\,(8\,709\,525\,d+5\,311\,735\,e\,x+3\,501\,225\,f\,x^2+2\,438\,667\,g\,x^3)\big)+143\,b^3\,x^7\,\big(229\,425\,c+17\,x\,(12\,075\,d+19\,x\,(575\,e+525\,f\,x+483\,g\,x^2))\big)+2\,a\,b^2\,x^4\,\big(29\,825\,250\,c+11\,x\,(2\,258\,025\,d+13\,x\,(148\,580\,e+21\,x\,(6175\,f+5474\,g\,x)))\big)\Big)+13\,935\,240\,(-1)^{2/3}\,3^{1/4}\,a^{11/3}\,(5\,b\,d-2\,a\,g)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-540\,i\,3^{3/4}\,a^{10/3}\,\left(39\,767\,(-b)^{1/3}\,b\,c+43\,010\,a^{1/3}\,b\,d-13\,832\,a\,(-b)^{1/3}\,f-17\,204\,a^{4/3}\,g\right)\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\Bigg)$$

Problem 459: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2\,(a+b\,x^3)^{3/2}\,(c+d\,x+e\,x^2+f\,x^3+g\,x^4)\,dx$$

Optimal (type 4, 742 leaves, 12 steps):

$$\frac{2 a^2 \left(7 b c-2 a f\right) \sqrt{a+b x^3}}{105 b^2}+\frac{54 a^2 \left(23 b d-8 a g\right) x \sqrt{a+b x^3}}{21505 b^2}+\frac{54 a^2 e x^2 \sqrt{a+b x^3}}{1729 b}+$$

$$\frac{2 a^2 f x^3 \sqrt{a+b x^3}}{105 b}+\frac{54 a^2 g x^4 \sqrt{a+b x^3}}{4301 b}-\frac{216 a^3 e \sqrt{a+b x^3}}{1729 b^{5/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\frac{1}{780045}$$

$$2 x^2 \left(a+b x^3\right)^{3/2} \left(52003 c x+45885 d x^2+41055 e x^3+37145 f x^4+33915 g x^5\right)+\frac{1}{111546435}$$

$$2 a x^2 \sqrt{a+b x^3} \left(7436429 c x+5368545 d x^2+4064445 e x^3+3187041 f x^4+2567565 g x^5\right)+$$

$$\left(108\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{10/3} e \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(1729 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$

$$\left(36\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a^3 \left(43010 \left(1-\sqrt{3}\right) a^{1/3} b^{2/3} e-1729 \left(23 b d-8 a g\right)\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(37182145 b^{7/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 436 leaves) :

$$\frac{1}{111\,546\,435\,(-b)^{8/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\,(a+b\,x^3)\,\left(-494\,a^3\,(4301\,f+2268\,g\,x)+\right.\right.$$

$$143\,b^3\,x^6\,(52\,003\,c+5\,x\,(9177\,d+17\,x\,(483\,e+437\,f\,x+399\,g\,x^2)))+$$
$$a^2\,b\,(7\,436\,429\,c+x\,(3\,221\,127\,d+x\,(1\,741\,905\,e+1\,062\,347\,f\,x+700\,245\,g\,x^2)))+$$
$$\left.2\,a\,b^2\,x^3\,(7\,436\,429\,c+x\,(5\,965\,050\,d+11\,x\,(451\,605\,e+247\,x\,(1564\,f+1365\,g\,x)))))\right)+$$

$$13\,935\,240\,(-1)^{2/3}\,3^{1/4}\,a^{11/3}\,b\,e\,\sqrt{(-1)^{5/6}\left(-1+\frac{(-b)^{1/3}\,x}{a^{1/3}}\right)}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-$$

$$108\,i\,3^{3/4}\,a^{10/3}\,\left(39\,767\,(-b)^{1/3}\,b\,d+43\,010\,a^{1/3}\,b\,e-13\,832\,a\,(-b)^{1/3}\,g\right)$$

$$\sqrt{\frac{(-1)^{5/6}\,\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\,\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 460: Result unnecessarily involves imaginary or complex numbers.

$$\int x\,(a+b\,x^3)^{3/2}\,(c+d\,x+e\,x^2+f\,x^3+g\,x^4)\,dx$$

Optimal (type 4, 723 leaves, 10 steps):

$$\frac{2 a^2 (7 b d-2 a g) \sqrt{a+b x^3}}{105 b^2}+\frac{54 a^2 e x \sqrt{a+b x^3}}{935 b}+\frac{54 a^2 f x^2 \sqrt{a+b x^3}}{1729 b}+\frac{2 a^2 g x^3 \sqrt{a+b x^3}}{105 b}+\frac{54 a^2 (19 b c-4 a f) \sqrt{a+b x^3}}{1729 b^{5/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+\frac{1}{440895}$$

$$2 x \left(a+b x^3\right)^{3/2} \left(33915 c x+29393 d x^2+25935 e x^3+23205 f x^4+20995 g x^5\right)+\frac{1}{4849845}$$

$$2 a x \sqrt{a+b x^3} \left(479655 c x+323323 d x^2+233415 e x^3+176715 f x^4+138567 g x^5\right)-$$

$$\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{7/3} (19 b c-4 a f) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(1729 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-$$

$$\left(18\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a^{7/3} \left(3458 a^{2/3} b^{1/3} e+935 \left(1-\sqrt{3}\right) (19 b c-4 a f)\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(1616615 b^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 429 leaves) :

$$\frac{1}{4849845\,(-b)^{8/3}\sqrt{a+b x^3}}\left(2\,(-b)^{2/3}\,(a+b x^3)\,\left(-92378\,a^3\,g+a^2\,b\,(323323\,d+x\,(140049\,e+187\,x\,(405\,f+247\,g\,x)))+\right.\right.$$
$$11\,b^3\,x^5\,\left(33915\,c+13\,x\,\left(2261\,d+5\,x\,\left(399\,e+357\,f\,x+323\,g\,x^2\right)\right)\right)+$$
$$\left.2\,a\,b^2\,x^2\,\left(426360\,c+x\,\left(323323\,d+x\,\left(259350\,e+215985\,f\,x+184756\,g\,x^2\right)\right)\right)\right)-$$
$$151470\,(-1)^{2/3}\,3^{1/4}\,a^{8/3}\,b\,(19\,b\,c-4\,a\,f)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$
$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]-$$
$$54\,i\,3^{3/4}\,a^{8/3}\,b\,\left(-17765\,b\,c+3458\,a^{2/3}\,(-b)^{1/3}\,e+3740\,a\,f\right)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$
$$\left.\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 461: Result unnecessarily involves imaginary or complex numbers.

$$\int (a+b x^3)^{3/2}\,(c+d x+e x^2+f x^3+g x^4)\,dx$$

Optimal (type 4, 694 leaves, 9 steps):

$$\frac{2a^2 e\sqrt{a+b x^3}}{15 b}+\frac{54 a^2 f x\sqrt{a+b x^3}}{935 b}+\frac{54 a^2 g x^2\sqrt{a+b x^3}}{1729 b}+\frac{54 a^2 (19 b d-4 a g)\sqrt{a+b x^3}}{1729 b^{5/3}\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)}+$$

$$\frac{1}{692835}2\left(a+b x^3\right)^{3/2}\left(62985 c x+53295 d x^2+46189 e x^3+40755 f x^4+36465 g x^5\right)+$$

$$\frac{1}{4849845}2 a\sqrt{a+b x^3}\left(793611 c x+479655 d x^2+323323 e x^3+233415 f x^4+176715 g x^5\right)-$$

$$\left(27\times 3^{1/4}\sqrt{2-\sqrt{3}}\,a^{7/3}(19 b d-4 a g)\left(a^{1/3}+b^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(1729 b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)+$$

$$\left(18\times 3^{3/4}\sqrt{2+\sqrt{3}}\,a^2\left(1729 b^{1/3}(17 b c-2 a f)-935\left(1-\sqrt{3}\right)a^{1/3}(19 b d-4 a g)\right)\left(a^{1/3}+b^{1/3}x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}b^{1/3}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x}\right],-7-4\sqrt{3}\right]\right)\Big/$$

$$\left(1616615 b^{5/3}\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}x\right)^2}}\sqrt{a+b x^3}\right)$$

Result (type 4, 429 leaves) :

$$-\frac{1}{4849845\,(-b)^{5/3}\sqrt{a+b\,x^3}}\left(2\,(-b)^{2/3}\,(a+b\,x^3)\,\left(a^2\,\left(323323\,e+81\,x\,(1729\,f+935\,g\,x)\right)+\right.\right.$$

$$7\,b^2\,x^4\,\left(62985\,c+11\,x\,\left(4845\,d+13\,x\,\left(323\,e+285\,f\,x+255\,g\,x^2\right)\right)\right)+$$

$$\left.2\,a\,b\,x\,\left(617253\,c+x\,\left(426360\,d+7\,x\,\left(46189\,e+37050\,f\,x+30855\,g\,x^2\right)\right)\right)\right)-$$

$$151470\,(-1)^{2/3}\,3^{1/4}\,a^{8/3}\,(19\,b\,d-4\,a\,g)\,\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}$$

$$\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]+$$

$$54\,i\,3^{3/4}\,a^{7/3}\,\left(323\,b\,\left(91\,(-b)^{1/3}\,c+55\,a^{1/3}\,d\right)-3458\,a\,(-b)^{1/3}\,f-3740\,a^{4/3}\,g\right)$$

$$\sqrt{\frac{(-1)^{5/6}\left(-a^{1/3}+(-b)^{1/3}\,x\right)}{a^{1/3}}}\;\sqrt{1+\frac{(-b)^{1/3}\,x}{a^{1/3}}+\frac{(-b)^{2/3}\,x^2}{a^{2/3}}}$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-(-1)^{5/6}-\frac{i\,(-b)^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,(-1)^{1/3}\right]\right)$$

Problem 462: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+b\,x^3)^{3/2}\,(c+d\,x+e\,x^2+f\,x^3+g\,x^4)}{x}\,dx$$

Optimal (type 4, 676 leaves, 12 steps):

$$\frac{2 a^2 f \sqrt{a+b x^3}}{15 b}+\frac{54 a^2 g x \sqrt{a+b x^3}}{935 b}+\frac{54 a^2 e \sqrt{a+b x^3}}{91 b^{2/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+$$

$$\frac{2 \left(a+b x^3\right)^{3/2} \left(12155 c x+9945 d x^2+8415 e x^3+7293 f x^4+6435 g x^5\right)}{109395 x}+\frac{1}{255255 x}$$

$$2 a \sqrt{a+b x^3} \left(85085 c x+41769 d x^2+25245 e x^3+17017 f x^4+12285 g x^5\right)-$$

$$\frac{2}{3} a^{3/2} c \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\; a^{7/3} e \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(91 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$

$$\left(18\times 3^{3/4} \sqrt{2+\sqrt{3}}\; a^2 \left(1547 b d-935 \left(1-\sqrt{3}\right) a^{1/3} b^{2/3} e-182 a g\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(85085 b^{4/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 753 leaves) :

$$\frac{1}{765765\,b}2\sqrt{a+b\,x^3}\left(273\,a^2\,(187\,f+81\,g\,x)+2\,a\,b\,\left(170170\,c+97461\,d\,x+67320\,e\,x^2+51051\,f\,x^3+40950\,g\,x^4\right)+7\,b^2\,x^3\,\left(12155\,c+9945\,d\,x+33\,x^2\,(255\,e+13\,x\,(17\,f+15\,g\,x))\right)\right)-$$

$$\frac{1}{255255\,b^{4/3}\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}}$$

$$2\,a^{3/2}\left(85085\,b^{4/3}\,c\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]+\right.$$

$$125307\,\sqrt{a}\,b\,d\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]-$$

$$14742\,a^{3/2}\,g\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\right],\,(-1)^{1/3}\right]-$$

$$75735\,\sqrt{2}\,a^{5/6}\,b^{2/3}\,e\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}$$

$$\sqrt{\frac{i\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}\left(-\left(-1+(-1)^{2/3}\right)\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

Problem 463: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(c+d x+e x^2+f x^3+g x^4\right)}{x^2} \, dx$$

Optimal (type 4, 692 leaves, 12 steps):

$$\frac{2 a^2 g \sqrt{a+b x^3}}{15 b}-\frac{27 a c \sqrt{a+b x^3}}{7 x}+\frac{27 a \left(13 b c+2 a f\right) \sqrt{a+b x^3}}{91 b^{2/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}+$$
$$\frac{2 a \sqrt{a+b x^3} \left(19305 c x+5005 d x^2+2457 e x^3+1485 f x^4+1001 g x^5\right)}{15015 x^2}+$$
$$\frac{2 \left(a+b x^3\right)^{3/2} \left(6435 c x+5005 d x^2+4095 e x^3+3465 f x^4+3003 g x^5\right)}{45045 x^2}-$$
$$\frac{2}{3} a^{3/2} d \,\text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{4/3} \left(13 b c+2 a f\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(182 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$
$$\left(9\times 3^{3/4} \sqrt{2+\sqrt{3}}\, a^{4/3} \left(182 a^{2/3} b^{1/3} e-55 \left(1-\sqrt{3}\right) \left(13 b c+2 a f\right)\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(5005 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 817 leaves):

$$\frac{1}{45045 b x}\sqrt{a+b x^3} \left(6006 a^2 g x+2 b^2 x^3 \left(6435 c+7 x \left(715 d+585 e x+495 f x^2+429 g x^3\right)\right)+\right.$$
$$\left.a b \left(-45045 c+4 x \left(10010 d+5733 e x+33 x^2 \left(120 f+91 g x\right)\right)\right)\right)-$$
$$\frac{1}{15015 b^{2/3} \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}}$$

$$a\left(10010\sqrt{a}\,b^{2/3}\,d\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{a+b\,x^3}\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]+\right.$$

$$14742\,a\,b^{1/3}\,e\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{a^{1/3}+b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}$$

$$\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}b^{1/3}x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],\,(-1)^{1/3}\right]-$$

$$57915\sqrt{2}\,a^{1/3}\,b\,c\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{\mathrm{i}\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3\,\mathrm{i}+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{\mathrm{i}\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\right.$$

$$\left.\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{\mathrm{i}\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)-$$

$$8910\sqrt{2}\,a^{4/3}\,f\left((-1)^{1/3}a^{1/3}-b^{1/3}x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}b^{1/3}x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{\mathrm{i}\left(1+\frac{b^{1/3}x}{a^{1/3}}\right)}{3\,\mathrm{i}+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{\mathrm{i}\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\right.$$

$$\left.\left.\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{\mathrm{i}\,b^{1/3}x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

Problem 464: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(c+d x+e x^2+f x^3+g x^4\right)}{x^3} \, dx$$

Optimal (type 4, 694 leaves, 11 steps):

$$\frac{27 a c \sqrt{a+b x^3}}{10 x^2}-\frac{27 a d \sqrt{a+b x^3}}{7 x}+\frac{27 a \left(13 b d+2 a g\right) \sqrt{a+b x^3}}{91 b^{2/3} \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-$$
$$\frac{1}{15015 x^3} 2 a \sqrt{a+b x^3} \left(27027 c x-19305 d x^2-5005 e x^3-2457 f x^4-1485 g x^5\right)+$$
$$\frac{2 \left(a+b x^3\right)^{3/2} \left(9009 c x+6435 d x^2+5005 e x^3+4095 f x^4+3465 g x^5\right)}{45045 x^3}-$$
$$\frac{2}{3} a^{3/2} e \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}} \, a^{4/3} \left(13 b d+2 a g\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(182 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$
$$\left(9\times 3^{3/4} \sqrt{2+\sqrt{3}} \, a \left(91 b^{1/3} \left(11 b c+4 a f\right)-110 \left(1-\sqrt{3}\right) a^{1/3} \left(13 b d+2 a g\right)\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$
$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$
$$\left(10010 b^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 952 leaves):

$$\frac{1}{90090 x^2} \sqrt{a+b x^3} \left(a \left(-45045 c-90090 d x+8 x^2 \left(10010 e+9 x \left(637 f+440 g x\right)\right)\right)+\right.$$
$$\left.4 b x^3 \left(9009 c+5 x \left(1287 d+7 x \left(143 e+117 f x+99 g x^2\right)\right)\right)\right)-$$
$$\frac{1}{30030 b^{2/3} \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}}$$

$$a\left(20020\sqrt{a}\,b^{2/3}\,e\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]+\right.$$

$$81081\,b^{4/3}\,c\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],\,(-1)^{1/3}\right]+$$

$$29484\,a\,b^{1/3}\,f\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\right],\,(-1)^{1/3}\right]-$$

$$115830\sqrt{2}\,a^{1/3}\,b\,d\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}$$

$$\sqrt{\frac{i\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)-$$

$$17820\sqrt{2}\,a^{4/3}\,g\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)a^{1/3}}}\,\sqrt{\frac{i\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]-\right.$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\right],\ \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\Bigg)\Bigg)$$

Problem 465: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b\,x^3\right)^{3/2}\left(c+d\,x+e\,x^2+f\,x^3+g\,x^4\right)}{x^4}\,dx$$

Optimal (type 4, 692 leaves, 12 steps):

$$\frac{a\,c\,\sqrt{a+b\,x^3}}{x^3}+\frac{27\,a\,d\,\sqrt{a+b\,x^3}}{10\,x^2}-\frac{27\,a\,e\,\sqrt{a+b\,x^3}}{7\,x}+\frac{27\,a\,b^{1/3}\,e\,\sqrt{a+b\,x^3}}{7\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)}-$$

$$\frac{2\,a\,\sqrt{a+b\,x^3}\,\left(1155\,c\,x+2079\,d\,x^2-1485\,e\,x^3-385\,f\,x^4-189\,g\,x^5\right)}{1155\,x^4}+$$

$$\frac{2\left(a+b\,x^3\right)^{3/2}\left(1155\,c\,x+693\,d\,x^2+495\,e\,x^3+385\,f\,x^4+315\,g\,x^5\right)}{3465\,x^4}-$$

$$\frac{1}{3}\sqrt{a}\,\left(3\,b\,c+2\,a\,f\right)\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-$$

$$\left(27\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,a^{4/3}\,b^{1/3}\,e\,\left(a^{1/3}+b^{1/3}\,x\right)\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticE}\left[\right.\right.$$

$$\left.\left.\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\ -7-4\sqrt{3}\right]\right)\Bigg/\left(14\,\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)+$$

$$\left(9\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,a\,\left(77\,b\,d-110\left(1-\sqrt{3}\right)a^{1/3}\,b^{2/3}\,e+28\,a\,g\right)\left(a^{1/3}+b^{1/3}\,x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x}\right],\ -7-4\sqrt{3}\right]\right)\Bigg/$$

$$\left(770\,b^{1/3}\,\sqrt{\frac{a^{1/3}\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)a^{1/3}+b^{1/3}\,x\right)^2}}\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 813 leaves):

$$\sqrt{a+b x^3}\left(a\left(\frac{8 f}{9}-\frac{c}{3 x^3}-\frac{d}{2 x^2}-\frac{e}{x}+\frac{28 g x}{55}\right)+b\left(\frac{2 c}{3}+\frac{2 d x}{5}+\frac{2 e x^2}{7}+\frac{2 f x^3}{9}+\frac{2 g x^4}{11}\right)\right)-$$

$$\sqrt{a}\ b\, c\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-\frac{2}{3} a^{3/2} f\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-$$

$$\left(27 a b^{2/3} d\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right.$$

$$\left.\sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]\right)\Big/$$

$$\left(10\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{a+b x^3}\right)-\left(54 a^2 g\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right.$$

$$\left.\sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]\right)\Big/$$

$$\left(55 b^{1/3}\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{a+b x^3}\right)-$$

$$\left(27\sqrt{2}\ a^{4/3} b^{1/3} e\left((-1)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{\frac{i\left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}\right.$$

$$\left(\left(-1+(-1)^{2/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\text{EllipticF}\left[\right.\right.$$

$$\left.\left.\left.\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\Big/\left(7\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{a+b x^3}\right)$$

Problem 466: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(a+b x^3\right)^{3/2}\left(c+d x+e x^2+f x^3+g x^4\right)}{x^5}\,dx$$

Optimal (type 4, 741 leaves, 13 steps):

$$\frac{27\,a\,c\,\sqrt{a+b\,x^3}}{20\,x^4}+\frac{a\,d\,\sqrt{a+b\,x^3}}{x^3}+\frac{27\,a\,e\,\sqrt{a+b\,x^3}}{10\,x^2}-\frac{27\,(7\,b\,c+8\,a\,f)\,\sqrt{a+b\,x^3}}{56\,x}+$$
$$\frac{27\,b^{1/3}\,(7\,b\,c+8\,a\,f)\,\sqrt{a+b\,x^3}}{56\,\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)}-\frac{2\,a\,\sqrt{a+b\,x^3}\,\left(189\,c\,x+105\,d\,x^2+189\,e\,x^3-135\,f\,x^4-35\,g\,x^5\right)}{105\,x^5}+$$
$$\frac{2\,\left(a+b\,x^3\right)^{3/2}\,\left(315\,c\,x+105\,d\,x^2+63\,e\,x^3+45\,f\,x^4+35\,g\,x^5\right)}{315\,x^5}-$$
$$\frac{1}{3}\,\sqrt{a}\,\,(3\,b\,d+2\,a\,g)\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-$$
$$\left(27\times 3^{1/4}\,\sqrt{2-\sqrt{3}}\,\,a^{1/3}\,b^{1/3}\,(7\,b\,c+8\,a\,f)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Bigg/\left(112\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)+$$
$$\left(9\times 3^{3/4}\,\sqrt{2+\sqrt{3}}\,\,a^{1/3}\,b^{1/3}\,\left(28\,a^{2/3}\,b^{1/3}\,e-5\,\left(1-\sqrt{3}\right)\,(7\,b\,c+8\,a\,f)\right)\,\left(a^{1/3}+b^{1/3}\,x\right)\,\sqrt{\frac{a^{2/3}-a^{1/3}\,b^{1/3}\,x+b^{2/3}\,x^2}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}{\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x}\right],\,-7-4\,\sqrt{3}\,\right]\right)\Bigg/\left(280\,\sqrt{\frac{a^{1/3}\,\left(a^{1/3}+b^{1/3}\,x\right)}{\left(\left(1+\sqrt{3}\right)\,a^{1/3}+b^{1/3}\,x\right)^2}}\,\,\sqrt{a+b\,x^3}\right)$$

Result (type 4, 878 leaves):

$$\frac{1}{2520 x^4}\sqrt{a+b x^3}\,\left(-70 a \left(9 c+2 x \left(6 d+x \left(9 e+2 x \left(9 f-8 g x\right)\right)\right)\right)+ b x^3 \left(-3465 c+16 x \left(105 d+x \left(63 e+5 x \left(9 f+7 g x\right)\right)\right)\right)\right)-$$

$$\sqrt{a}\, b\, d\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-\frac{2}{3} a^{3/2} g\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-$$

$$\left(27 a b^{2/3} e \left(\left(-1\right)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}}\sqrt{\frac{\left(-1\right)^{1/3} a^{1/3}-\left(-1\right)^{2/3} b^{1/3} x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3} b^{1/3} x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}}\right],\left(-1\right)^{1/3}\right]\right)\Big/\left(10\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3} b^{1/3} x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}}\sqrt{a+b x^3}\right)-$$

$$\left(27 a^{1/3} b^{4/3} c \left(\left(-1\right)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{\left(-1\right)^{1/3} a^{1/3}-\left(-1\right)^{2/3} b^{1/3} x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}}\sqrt{\frac{i \left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}\right.$$

$$\left(\left(-1+\left(-1\right)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-1\right)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{\left(-1\right)^{1/3}}{-1+\left(-1\right)^{1/3}}\right]+\text{EllipticF}\left[\right.\right.$$

$$\left.\left.\left.\text{ArcSin}\left[\frac{\sqrt{\left(-1\right)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{\left(-1\right)^{1/3}}{-1+\left(-1\right)^{1/3}}\right]\right)\right)\Big/\left(4\sqrt{2}\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3} b^{1/3} x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}}\sqrt{a+b x^3}\right)-$$

$$\left(27\sqrt{2}\, a^{4/3} b^{1/3} f \left(\left(-1\right)^{1/3} a^{1/3}-b^{1/3} x\right)\sqrt{\frac{\left(-1\right)^{1/3} a^{1/3}-\left(-1\right)^{2/3} b^{1/3} x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}}\sqrt{\frac{i \left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}\right.$$

$$\left(\left(-1+\left(-1\right)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\left(-1\right)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{\left(-1\right)^{1/3}}{-1+\left(-1\right)^{1/3}}\right]+\text{EllipticF}\left[\right.\right.$$

$$\left.\left.\left.\text{ArcSin}\left[\frac{\sqrt{\left(-1\right)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{\left(-1\right)^{1/3}}{-1+\left(-1\right)^{1/3}}\right]\right)\right)\Big/\left(7\sqrt{\frac{a^{1/3}+\left(-1\right)^{2/3} b^{1/3} x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}}\sqrt{a+b x^3}\right)$$

Problem 467: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(c+d x+e x^2+f x^3+g x^4\right)}{x^6} \, dx$$

Optimal (type 4, 689 leaves, 11 steps):

$$\frac{27 b c \sqrt{a+b x^3}}{20 x^2}-\frac{27 b d \sqrt{a+b x^3}}{8 x}+\frac{27 b^{1/3} (7 b d+8 a g) \sqrt{a+b x^3}}{56 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{1}{60} \left(\frac{12 c}{x^5}+\frac{15 d}{x^4}+\frac{20 e}{x^3}+\frac{30 f}{x^2}+\frac{60 g}{x}\right) \left(a+b x^3\right)^{3/2}-\frac{b \sqrt{a+b x^3} \left(252 c x-315 d x^2-140 e x^3-126 f x^4-180 g x^5\right)}{140 x^3}-\sqrt{a}\, b e \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\, a^{1/3} b^{1/3} (7 b d+8 a g) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/\left(112 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+\left(9\times 3^{3/4} \sqrt{2+\sqrt{3}}\, b^{1/3} \left(14 b^{1/3} (b c+2 a f)-5 \left(1-\sqrt{3}\right) a^{1/3} (7 b d+8 a g)\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/\left(280 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 949 leaves):

$$-\frac{1}{840 x^5}\sqrt{a+b x^3} \left(14 a \left(12 c+5 x \left(3 d+4 e x+6 x^2 (f+2 g x)\right)\right)+b x^3 \left(546 c+x \left(1155 d-16 x \left(35 e+3 x (7 f+5 g x)\right)\right)\right)\right)-\frac{1}{280 \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}}$$

$$b^{1/3} \left(280 \sqrt{a}\, b^{2/3} e \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]+\right.$$

$$378\, b^{4/3}\, c \left((-1)^{1/3} a^{1/3} - b^{1/3} x\right) \sqrt{\frac{a^{1/3} + b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{(-1)^{1/3} \left(a^{1/3} - (-1)^{1/3} b^{1/3} x\right)}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}\right], (-1)^{1/3}\right] +$$

$$756\, a\, b^{1/3}\, f \left((-1)^{1/3} a^{1/3} - b^{1/3} x\right) \sqrt{\frac{a^{1/3} + b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{(-1)^{1/3} \left(a^{1/3} - (-1)^{1/3} b^{1/3} x\right)}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}\right], (-1)^{1/3}\right] -$$

$$945 \sqrt{2}\, a^{1/3}\, b\, d \left((-1)^{1/3} a^{1/3} - b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3} \left(a^{1/3} - (-1)^{1/3} b^{1/3} x\right)}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{i \left(1 + \frac{b^{1/3} x}{a^{1/3}}\right)}{3 i + \sqrt{3}}}$$

$$\left(-\left(-1 + (-1)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1 + (-1)^{1/3}}\right] -\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1 + (-1)^{1/3}}\right]\right) -$$

$$1080 \sqrt{2}\, a^{4/3}\, g \left((-1)^{1/3} a^{1/3} - b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3} \left(a^{1/3} - (-1)^{1/3} b^{1/3} x\right)}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{i \left(1 + \frac{b^{1/3} x}{a^{1/3}}\right)}{3 i + \sqrt{3}}}$$

$$\left(-\left(-1 + (-1)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1 + (-1)^{1/3}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1 + (-1)^{1/3}}\right]\right)\right)$$

Problem 468: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(c+d x+e x^2+f x^3+g x^4\right)}{x^7} \, dx$$

Optimal (type 4, 692 leaves, 12 steps):

$$\frac{b c \sqrt{a+b x^3}}{4 x^3}+\frac{27 b d \sqrt{a+b x^3}}{20 x^2}-\frac{27 b e \sqrt{a+b x^3}}{8 x}+\frac{27 b^{4/3} e \sqrt{a+b x^3}}{8 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{1}{60} \left(\frac{10 c}{x^6}+\frac{12 d}{x^5}+\frac{15 e}{x^4}+\frac{20 f}{x^3}+\frac{30 g}{x^2}\right) \left(a+b x^3\right)^{3/2}-\frac{b \sqrt{a+b x^3} \left(10 c x+36 d x^2-45 e x^3-20 f x^4-18 g x^5\right)}{20 x^4}-\frac{b \left(b c+4 a f\right) \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{4 \sqrt{a}}-\left(27 \times 3^{1/4} \sqrt{2-\sqrt{3}}\; a^{1/3} b^{4/3} e \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/ \left(16 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+\left(9 \times 3^{3/4} \sqrt{2+\sqrt{3}}\; b^{2/3} \left(2 b d-5 \left(1-\sqrt{3}\right) a^{1/3} b^{2/3} e+4 a g\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/ \left(40 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 805 leaves):

$$-\frac{1}{120 x^6}\sqrt{a+b x^3}\,\left(b x^3 \left(50 c+x \left(78 d+x \left(165 e-80 f x-48 g x^2\right)\right)\right)+a \left(20 c+2 x \left(12 d+5 x \left(3 e+4 f x+6 g x^2\right)\right)\right)\right)+$$

$$\frac{3}{80} b \left(-\frac{20 b c \,\text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{3 \sqrt{a}}-\frac{80}{3} \sqrt{a}\, f \,\text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-\right.$$

$$\left(36 b^{2/3} d \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\,\text{EllipticF}\left[\right.\right.$$

$$\left.\left.\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]\right)\Big/\left(\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}\right)-$$

$$\left(72 a g \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{a^{1/3}+b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\,\text{EllipticF}\left[\right.\right.$$

$$\left.\left.\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]\right)\Big/\left(b^{1/3}\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}\right)-$$

$$\left(90 \sqrt{2}\, a^{1/3} b^{1/3} e \left((-1)^{1/3} a^{1/3}-b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3} a^{1/3}-(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{i \left(1+\frac{b^{1/3} x}{a^{1/3}}\right)}{3 i+\sqrt{3}}}\right.$$

$$\left(\left(-1+(-1)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\text{EllipticF}\left[\right.\right.$$

$$\left.\left.\left.\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)\Big/\left(\sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}\right)\Bigg)$$

Problem 469: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(c+d x+e x^2+f x^3+g x^4\right)}{x^8}\, dx$$

Optimal (type 4, 746 leaves, 13 steps):

$$\frac{27 b c \sqrt{a+b x^3}}{280 x^4}+\frac{b d \sqrt{a+b x^3}}{4 x^3}+\frac{27 b e \sqrt{a+b x^3}}{20 x^2}-\frac{27 b \left(b c+14 a f\right) \sqrt{a+b x^3}}{112 a x}+$$

$$\frac{27 b^{4/3} \left(b c+14 a f\right) \sqrt{a+b x^3}}{112 a \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{1}{420}\left(\frac{60 c}{x^7}+\frac{70 d}{x^6}+\frac{84 e}{x^5}+\frac{105 f}{x^4}+\frac{140 g}{x^3}\right)\left(a+b x^3\right)^{3/2}-$$

$$\frac{b \sqrt{a+b x^3} \left(36 c x+70 d x^2+252 e x^3-315 f x^4-140 g x^5\right)}{140 x^5}-$$

$$\frac{b \left(b d+4 a g\right) \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{4 \sqrt{a}}-\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\, b^{4/3} \left(b c+14 a f\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(224 a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \sqrt{a+b x^3}\right)+$$

$$\left(9\times 3^{3/4} \sqrt{2+\sqrt{3}}\, b^{4/3} \left(28 a^{2/3} b^{1/3} e-5 \left(1-\sqrt{3}\right) \left(b c+14 a f\right)\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Big/$$

$$\left(560 a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \sqrt{a+b x^3}\right)$$

Result (type 4, 897 leaves):

$$-\frac{1}{1680 a x^7}\sqrt{a+b x^3} \left(405 b^2 c x^6+2 a b x^3 \left(255 c+7 x \left(50 d+x \left(78 e+165 f x-80 g x^2\right)\right)\right)+\right.$$

$$\left.4 a^2 \left(60 c+7 x \left(10 d+x \left(12 e+5 x \left(3 f+4 g x\right)\right)\right)\right)\right)-\frac{1}{560 a \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{a+b x^3}}$$

$$b \left(140 \sqrt{a}\, b d \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\ \sqrt{a+b x^3}\ \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]+\right.$$

$$560\, a^{3/2}\, g \sqrt{\frac{a^{1/3}+(-1)^{2/3}\, b^{1/3}\, x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \sqrt{a+b\, x^3}\, \text{ArcTanh}\left[\frac{\sqrt{a+b\, x^3}}{\sqrt{a}}\right] +$$

$$756\, a\, b^{2/3}\, e \left((-1)^{1/3}\, a^{1/3}-b^{1/3}\, x\right) \sqrt{\frac{a^{1/3}+b^{1/3}\, x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}\, b^{1/3}\, x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\, b^{1/3}\, x}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\right],\, (-1)^{1/3}\right] -$$

$$135\sqrt{2}\, a^{1/3}\, b^{4/3}\, c \left((-1)^{1/3}\, a^{1/3}-b^{1/3}\, x\right) \sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}\, b^{1/3}\, x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}\, \sqrt{\frac{i\left(1+\frac{b^{1/3}\, x}{a^{1/3}}\right)}{3\, i+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\, b^{1/3}\, x}{a^{1/3}}}}{3^{1/4}}\right],\, \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right] -\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\, b^{1/3}\, x}{a^{1/3}}}}{3^{1/4}}\right],\, \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right) -$$

$$1890\sqrt{2}\, a^{4/3}\, b^{1/3}\, f \left((-1)^{1/3}\, a^{1/3}-b^{1/3}\, x\right) \sqrt{\frac{(-1)^{1/3}\left(a^{1/3}-(-1)^{1/3}\, b^{1/3}\, x\right)}{\left(1+(-1)^{1/3}\right) a^{1/3}}}$$

$$\sqrt{\frac{i\left(1+\frac{b^{1/3}\, x}{a^{1/3}}\right)}{3\, i+\sqrt{3}}} \left(-\left(-1+(-1)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\, b^{1/3}\, x}{a^{1/3}}}}{3^{1/4}}\right],\, \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right] -\right.$$

$$\left.\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i\, b^{1/3}\, x}{a^{1/3}}}}{3^{1/4}}\right],\, \frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\right)\right)$$

Problem 470: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b\, x^3\right)^{3/2} \left(c+d\, x+e\, x^2+f\, x^3+g\, x^4\right)}{x^9}\, dx$$

Optimal (type 4, 705 leaves, 11 steps):

$$-\frac{1}{560} b \left(\frac{63 c}{x^5}+\frac{90 d}{x^4}+\frac{140 e}{x^3}+\frac{252 f}{x^2}+\frac{630 g}{x}\right) \sqrt{a+b x^3}-\frac{27 b^2 c \sqrt{a+b x^3}}{320 a x^2}-\frac{27 b^2 d \sqrt{a+b x^3}}{112 a x}+\frac{27 b^{4/3} (b d+14 a g) \sqrt{a+b x^3}}{112 a \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-$$

$$\frac{1}{840} \left(\frac{105 c}{x^8}+\frac{120 d}{x^7}+\frac{140 e}{x^6}+\frac{168 f}{x^5}+\frac{210 g}{x^4}\right) \left(a+b x^3\right)^{3/2}-\frac{b^2 e \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{4 \sqrt{a}}-$$

$$\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\, b^{4/3} (b d+14 a g) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(224 a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-$$

$$\left(9\times 3^{3/4} \sqrt{2+\sqrt{3}}\, b^{4/3} \left(7 b^{1/3} (b c-16 a f)+20 \left(1-\sqrt{3}\right) a^{1/3} (b d+14 a g)\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(2240 a \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 978 leaves):

$$-\frac{1}{6720 a x^8}$$

$$\sqrt{a+b x^3} \left(81 b^2 x^6 (7 c+20 d x)+4 a b x^3 \left(399 c+2 x \left(255 d+7 x \left(50 e+78 f x+165 g x^2\right)\right)\right)+\right.$$

$$\left.8 a^2 (105 c+2 x (60 d+7 x (10 e+3 x (4 f+5 g x))))\right)-\frac{1}{2240 a \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}}$$

$$b^{4/3} \left(560 \sqrt{a}\, b^{2/3} e \sqrt{\frac{a^{1/3}+(-1)^{2/3} b^{1/3} x}{\left(1+(-1)^{1/3}\right) a^{1/3}}} \sqrt{a+b x^3}\, \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]-\right.$$

$$189\, b^{4/3}\, c\, \left((-1)^{1/3} a^{1/3} - b^{1/3} x\right) \sqrt{\frac{a^{1/3} + b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{(-1)^{1/3} \left(a^{1/3} - (-1)^{1/3} b^{1/3} x\right)}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}\right], (-1)^{1/3}\right] +$$

$$3024\, a\, b^{1/3}\, f\, \left((-1)^{1/3} a^{1/3} - b^{1/3} x\right) \sqrt{\frac{a^{1/3} + b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{(-1)^{1/3} \left(a^{1/3} - (-1)^{1/3} b^{1/3} x\right)}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{a^{1/3} + (-1)^{2/3} b^{1/3} x}{\left(1 + (-1)^{1/3}\right) a^{1/3}}}\right], (-1)^{1/3}\right] -$$

$$540 \sqrt{2}\, a^{1/3}\, b\, d\, \left((-1)^{1/3} a^{1/3} - b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3} \left(a^{1/3} - (-1)^{1/3} b^{1/3} x\right)}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{i \left(1 + \frac{b^{1/3} x}{a^{1/3}}\right)}{3 i + \sqrt{3}}}$$

$$\left(-\left(-1 + (-1)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1 + (-1)^{1/3}}\right] - \right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1 + (-1)^{1/3}}\right]\right) -$$

$$7560 \sqrt{2}\, a^{4/3}\, g\, \left((-1)^{1/3} a^{1/3} - b^{1/3} x\right) \sqrt{\frac{(-1)^{1/3} \left(a^{1/3} - (-1)^{1/3} b^{1/3} x\right)}{\left(1 + (-1)^{1/3}\right) a^{1/3}}} \sqrt{\frac{i \left(1 + \frac{b^{1/3} x}{a^{1/3}}\right)}{3 i + \sqrt{3}}}$$

$$\left(-\left(-1 + (-1)^{2/3}\right) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1 + (-1)^{1/3}}\right] - \right.$$

$$\left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6} - \frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right], \frac{(-1)^{1/3}}{-1 + (-1)^{1/3}}\right]\right)\right)$$

Problem 471: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(c+d x+e x^2+f x^3+g x^4\right)}{x^{10}} \, dx$$

Optimal (type 4, 714 leaves, 12 steps):

$$-\frac{b \left(\frac{140 c}{x^6}+\frac{189 d}{x^5}+\frac{270 e}{x^4}+\frac{420 f}{x^3}+\frac{756 g}{x^2}\right) \sqrt{a+b x^3}}{1680}-\frac{b^2 c \sqrt{a+b x^3}}{24 a x^3}-\frac{27 b^2 d \sqrt{a+b x^3}}{320 a x^2}-\frac{27 b^2 e \sqrt{a+b x^3}}{112 a x}+\frac{27 b^{7/3} e \sqrt{a+b x^3}}{112 a \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{\left(\frac{280 c}{x^9}+\frac{315 d}{x^8}+\frac{360 e}{x^7}+\frac{420 f}{x^6}+\frac{504 g}{x^5}\right) \left(a+b x^3\right)^{3/2}}{2520}+\frac{b^2 \left(b c-6 a f\right) \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{24 a^{3/2}}-$$

$$\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\; b^{7/3} e \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/ \left(224 a^{2/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)-$$

$$\left(9\times 3^{3/4} \sqrt{2+\sqrt{3}}\; b^{5/3} \left(7 b d+20 \left(1-\sqrt{3}\right) a^{1/3} b^{2/3} e-112 a g\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right) \Big/ \left(2240 a \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 1056 leaves):

$$\left(-\frac{a c}{9 x^9}-\frac{a d}{8 x^8}-\frac{a e}{7 x^7}+\frac{-7 b c-6 a f}{36 x^6}+\frac{-19 b d-16 a g}{80 x^5}-\frac{17 b e}{56 x^4}-\frac{b \left(b c+10 a f\right)}{24 a x^3}-\frac{b \left(27 b d+208 a g\right)}{320 a x^2}-\frac{27 b^2 e}{112 a x}\right) \sqrt{a+b x^3}+$$

$$\frac{1}{4480\,a}\,b^2\,\Bigg(\frac{560\,b\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]}{3\,\sqrt{a}}-1120\,\sqrt{a}\,f\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\right]-$$

$$\Bigg(378\,b\,d\,\sqrt{\frac{\frac{a^{1/3}}{b^{1/3}}+x}{\frac{a^{1/3}}{b^{1/3}}+\frac{(-1)^{1/3}\,a^{1/3}}{b^{1/3}}}}\,\left(-\frac{(-1)^{1/3}\,a^{1/3}}{b^{1/3}}+x\right)\sqrt{\frac{\frac{(-1)^{2/3}\,a^{1/3}}{b^{1/3}}+x}{\frac{(-1)^{1/3}\,a^{1/3}}{b^{1/3}}+\frac{(-1)^{2/3}\,a^{1/3}}{b^{1/3}}}}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-b^{1/3}\,x}{\left((-1)^{1/3}+(-1)^{2/3}\right)a^{1/3}}}\right],\,(-1)^{1/3}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{\frac{-\frac{(-1)^{1/3}\,a^{1/3}}{b^{1/3}}+x}{-\frac{(-1)^{1/3}\,a^{1/3}}{b^{1/3}}-\frac{(-1)^{2/3}\,a^{1/3}}{b^{1/3}}}}\,\sqrt{a+b\,x^3}\right)+\Bigg(6048\,a\,g\,\sqrt{\frac{\frac{a^{1/3}}{b^{1/3}}+x}{\frac{a^{1/3}}{b^{1/3}}+\frac{(-1)^{1/3}\,a^{1/3}}{b^{1/3}}}}\,\left(-\frac{(-1)^{1/3}\,a^{1/3}}{b^{1/3}}+x\right)$$

$$\sqrt{\frac{\frac{(-1)^{2/3}\,a^{1/3}}{b^{1/3}}+x}{\frac{(-1)^{1/3}\,a^{1/3}}{b^{1/3}}+\frac{(-1)^{2/3}\,a^{1/3}}{b^{1/3}}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{1/3}\,a^{1/3}-b^{1/3}\,x}{\left((-1)^{1/3}+(-1)^{2/3}\right)a^{1/3}}}\right],\,(-1)^{1/3}\right]\Bigg)\Bigg/$$

$$\left(\sqrt{\frac{-\frac{(-1)^{1/3}\,a^{1/3}}{b^{1/3}}+x}{-\frac{(-1)^{1/3}\,a^{1/3}}{b^{1/3}}-\frac{(-1)^{2/3}\,a^{1/3}}{b^{1/3}}}}\,\sqrt{a+b\,x^3}\right)+$$

$$\Bigg(1080\,b\,e\,\sqrt{-\frac{b^{1/3}\left(\frac{a^{1/3}}{b^{1/3}}+x\right)}{-a^{1/3}+(-1)^{2/3}\,a^{1/3}}}\,\left(-\frac{(-1)^{1/3}\,a^{1/3}}{b^{1/3}}+x\right)\sqrt{\frac{\frac{(-1)^{2/3}\,a^{1/3}}{b^{1/3}}+x}{\frac{(-1)^{1/3}\,a^{1/3}}{b^{1/3}}+\frac{(-1)^{2/3}\,a^{1/3}}{b^{1/3}}}}$$

$$\Bigg(\frac{1}{b^{1/3}}\left(-a^{1/3}+(-1)^{2/3}\,a^{1/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{2/3}\,a^{1/3}+b^{1/3}\,x}{\left((-1)^{1/3}+(-1)^{2/3}\right)a^{1/3}}}\right],\right.$$

$$\left.\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\frac{a^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{2/3}\,a^{1/3}+b^{1/3}\,x}{\left((-1)^{1/3}+(-1)^{2/3}\right)a^{1/3}}}\right],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]}{b^{1/3}}\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{\frac{-\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}}+x}{-\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}}-\frac{(-1)^{2/3} a^{1/3}}{b^{1/3}}}}\ \sqrt{a+b x^3}\right)\Bigg)$$

Problem 472: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(c+d x+e x^2+f x^3+g x^4\right)}{x^{11}} \, dx$$

Optimal (type 4, 764 leaves, 13 steps):

$$-\frac{b \left(\frac{108 c}{x^7}+\frac{140 d}{x^6}+\frac{189 e}{x^5}+\frac{270 f}{x^4}+\frac{420 g}{x^3}\right) \sqrt{a+b x^3}}{1680}-\frac{27 b^2 c \sqrt{a+b x^3}}{1120 a x^4}-\frac{b^2 d \sqrt{a+b x^3}}{24 a x^3}-\frac{27 b^2 e \sqrt{a+b x^3}}{320 a x^2}+\frac{27 b^2 (b c-4 a f) \sqrt{a+b x^3}}{448 a^2 x}-\frac{27 b^{7/3} (b c-4 a f) \sqrt{a+b x^3}}{448 a^2 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{\left(\frac{252 c}{x^{10}}+\frac{280 d}{x^9}+\frac{315 e}{x^8}+\frac{360 f}{x^7}+\frac{420 g}{x^6}\right) \left(a+b x^3\right)^{3/2}}{2520}+\frac{b^2 (b d-6 a g) \text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{24 a^{3/2}}+\left(27\times 3^{1/4} \sqrt{2-\sqrt{3}}\ b^{7/3} (b c-4 a f) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/\left(896 a^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \sqrt{a+b x^3}\right)-\left(9\times 3^{3/4} \sqrt{2+\sqrt{3}}\ b^{7/3} \left(7 a^{2/3} b^{1/3} e-5 \left(1-\sqrt{3}\right) (b c-4 a f)\right) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/\left(2240 a^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\ \sqrt{a+b x^3}\right)$$

Result (type 4, 930 leaves):

$$-\frac{1}{20160 a^2 x^{10}}\sqrt{a+b x^3} \left(-1215 b^3 c x^9+8 a^3 \left(252 c+5 x \left(56 d+63 e x+72 f x^2+84 g x^3\right)\right)+\right.$$

$$3\,a\,b^2\,x^6\,\left(162\,c+x\,\left(280\,d+81\,x\,\left(7\,e+20\,f\,x\right)\right)\right)+$$
$$4\,a^2\,b\,x^3\,\left(828\,c+x\,\left(980\,d+3\,x\,\left(399\,e+510\,f\,x+700\,g\,x^2\right)\right)\right)\Big)+$$

$$\frac{1}{6720\,a^2\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}}\,b^2\,\left(280\,\sqrt{a}\,b\,d\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\right.$$

$$\text{ArcTanh}\Big[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\Big]-1680\,a^{3/2}\,g\,\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{a+b\,x^3}\,\text{ArcTanh}\Big[\frac{\sqrt{a+b\,x^3}}{\sqrt{a}}\Big]+$$

$$567\,a\,b^{2/3}\,e\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{a^{1/3}+b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}$$

$$\text{EllipticF}\Big[\text{ArcSin}\Big[\sqrt{\frac{a^{1/3}+(-1)^{2/3}\,b^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\Big],\,(-1)^{1/3}\Big]-$$

$$405\,\sqrt{2}\,a^{1/3}\,b^{4/3}\,c\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}\,\sqrt{\frac{\mathrm{i}\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,\mathrm{i}+\sqrt{3}}}$$

$$\left(-\left(-1+(-1)^{2/3}\right)\,\text{EllipticE}\Big[\text{ArcSin}\Big[\frac{\sqrt{(-1)^{1/6}-\frac{\mathrm{i}\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\Big],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\Big]-\right.$$

$$\left.\text{EllipticF}\Big[\text{ArcSin}\Big[\frac{\sqrt{(-1)^{1/6}-\frac{\mathrm{i}\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\Big],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\Big]\right)+$$

$$1620\,\sqrt{2}\,a^{4/3}\,b^{1/3}\,f\,\left((-1)^{1/3}\,a^{1/3}-b^{1/3}\,x\right)\,\sqrt{\frac{(-1)^{1/3}\,\left(a^{1/3}-(-1)^{1/3}\,b^{1/3}\,x\right)}{\left(1+(-1)^{1/3}\right)\,a^{1/3}}}$$

$$\sqrt{\frac{\mathrm{i}\,\left(1+\frac{b^{1/3}\,x}{a^{1/3}}\right)}{3\,\mathrm{i}+\sqrt{3}}}\,\left(-\left(-1+(-1)^{2/3}\right)\,\text{EllipticE}\Big[\text{ArcSin}\Big[\frac{\sqrt{(-1)^{1/6}-\frac{\mathrm{i}\,b^{1/3}\,x}{a^{1/3}}}}{3^{1/4}}\Big],\,\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\Big]-\right.$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{(-1)^{1/6}-\frac{i b^{1/3} x}{a^{1/3}}}}{3^{1/4}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]\Bigg)\Bigg)$$

Problem 473: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^3\right)^{3/2} \left(c+d x+e x^2+f x^3+g x^4\right)}{x^{12}}\, dx$$

Optimal (type 4, 796 leaves, 14 steps):

$$-\frac{b \left(\frac{945 c}{x^8}+\frac{1188 d}{x^7}+\frac{1540 e}{x^6}+\frac{2079 f}{x^5}+\frac{2970 g}{x^4}\right) \sqrt{a+b x^3}}{18480}-\frac{27 b^2 c \sqrt{a+b x^3}}{1760 a x^5}-\frac{27 b^2 d \sqrt{a+b x^3}}{1120 a x^4}-\frac{b^2 e \sqrt{a+b x^3}}{24 a x^3}+\frac{27 b^2 (7 b c-22 a f) \sqrt{a+b x^3}}{7040 a^2 x^2}+\frac{27 b^2 (b d-4 a g) \sqrt{a+b x^3}}{448 a^2 x}-\frac{27 b^{7/3} (b d-4 a g) \sqrt{a+b x^3}}{448 a^2 \left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)}-\frac{\left(\frac{2520 c}{x^{11}}+\frac{2772 d}{x^{10}}+\frac{3080 e}{x^9}+\frac{3465 f}{x^8}+\frac{3960 g}{x^7}\right) \left(a+b x^3\right)^{3/2}}{27720}+\frac{b^3 e \,\text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]}{24 a^{3/2}}+$$

$$\left(27 \times 3^{1/4} \sqrt{2-\sqrt{3}}\, b^{7/3} (b d-4 a g) \left(a^{1/3}+b^{1/3} x\right) \sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(896 a^{5/3} \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)+$$

$$\left(9 \times 3^{3/4} \sqrt{2+\sqrt{3}}\, b^{7/3} \left(7 b^{1/3} (7 b c-22 a f)+110 \left(1-\sqrt{3}\right) a^{1/3} (b d-4 a g)\right) \left(a^{1/3}+b^{1/3} x\right)\right.$$

$$\left.\sqrt{\frac{a^{2/3}-a^{1/3} b^{1/3} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\left(1-\sqrt{3}\right) a^{1/3}+b^{1/3} x}{\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x}\right],-7-4 \sqrt{3}\right]\right)\Bigg/$$

$$\left(49280 a^2 \sqrt{\frac{a^{1/3} \left(a^{1/3}+b^{1/3} x\right)}{\left(\left(1+\sqrt{3}\right) a^{1/3}+b^{1/3} x\right)^2}} \sqrt{a+b x^3}\right)$$

Result (type 4, 1427 leaves) :

$$\left(-\frac{a c}{11 x^{11}}-\frac{a d}{10 x^{10}}-\frac{a e}{9 x^9}+\frac{-25 b c-22 a f}{176 x^8}+\frac{-23 b d-20 a g}{140 x^7}-\frac{7 b e}{36 x^6}-\frac{b\left(27 b c+418 a f\right)}{1760 a x^5}-\frac{b\left(27 b d+340 a g\right)}{1120 a x^4}-\frac{b^2 e}{24 a x^3}-\frac{27 b^2\left(-7 b c+22 a f\right)}{7040 a^2 x^2}-\frac{27 b^2\left(-b d+4 a g\right)}{448 a^2 x}\right)\sqrt{a+b x^3}+$$

$$\frac{1}{98560 a^2} b^3 \left(\frac{12320}{3}\sqrt{a}\, e \,\text{ArcTanh}\left[\frac{\sqrt{a+b x^3}}{\sqrt{a}}\right]+\left(2646 b c \sqrt{\frac{\frac{a^{1/3}}{b^{1/3}}+x}{\frac{a^{1/3}}{b^{1/3}}+\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}}}}\left(-\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}}+x\right)\right.\right.$$

$$\left.\sqrt{\frac{\frac{(-1)^{2/3} a^{1/3}}{b^{1/3}}+x}{\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}}+\frac{(-1)^{2/3} a^{1/3}}{b^{1/3}}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{1/3} a^{1/3}-b^{1/3} x}{\left((-1)^{1/3}+(-1)^{2/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]\right)\Big/$$

$$\left(\sqrt{\frac{-\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}}+x}{-\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}}-\frac{(-1)^{2/3} a^{1/3}}{b^{1/3}}}}\sqrt{a+b x^3}\right)-\left(8316 a f\sqrt{\frac{\frac{a^{1/3}}{b^{1/3}}+x}{\frac{a^{1/3}}{b^{1/3}}+\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}}}}\left(-\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}}+x\right)\right.$$

$$\left.\sqrt{\frac{\frac{(-1)^{2/3} a^{1/3}}{b^{1/3}}+x}{\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}}+\frac{(-1)^{2/3} a^{1/3}}{b^{1/3}}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{1/3} a^{1/3}-b^{1/3} x}{\left((-1)^{1/3}+(-1)^{2/3}\right) a^{1/3}}}\right],(-1)^{1/3}\right]\right)\Big/$$

$$\left(\sqrt{\frac{-\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}}+x}{-\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}}-\frac{(-1)^{2/3} a^{1/3}}{b^{1/3}}}}\sqrt{a+b x^3}\right)-$$

$$\left(5940 b d\sqrt{-\frac{b^{1/3}\left(\frac{a^{1/3}}{b^{1/3}}+x\right)}{-a^{1/3}+(-1)^{2/3} a^{1/3}}}\left(-\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}}+x\right)\sqrt{\frac{\frac{(-1)^{2/3} a^{1/3}}{b^{1/3}}+x}{\frac{(-1)^{1/3} a^{1/3}}{b^{1/3}}+\frac{(-1)^{2/3} a^{1/3}}{b^{1/3}}}}\right.$$

$$\left(\frac{1}{b^{1/3}}\left(-a^{1/3}+(-1)^{2/3} a^{1/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{2/3} a^{1/3}+b^{1/3} x}{\left((-1)^{1/3}+(-1)^{2/3}\right) a^{1/3}}}\right],\right.\right.$$

$$\left.\left.\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]+\frac{a^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{2/3} a^{1/3}+b^{1/3} x}{\left((-1)^{1/3}+(-1)^{2/3}\right) a^{1/3}}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]}{b^{1/3}}\right)\right)\Big/$$

$$\left(\sqrt{\frac{-\frac{(-1)^{1/3}a^{1/3}}{b^{1/3}}+x}{-\frac{(-1)^{1/3}a^{1/3}}{b^{1/3}}-\frac{(-1)^{2/3}a^{1/3}}{b^{1/3}}}}\sqrt{a+b\,x^3}\right)+\left(23\,760\,a\,g\sqrt{-\frac{b^{1/3}\left(\frac{a^{1/3}}{b^{1/3}}+x\right)}{-a^{1/3}+\left(-1\right)^{2/3}a^{1/3}}}\right.$$

$$\left(-\frac{\left(-1\right)^{1/3}a^{1/3}}{b^{1/3}}+x\right)\sqrt{\frac{\frac{(-1)^{2/3}a^{1/3}}{b^{1/3}}+x}{\frac{(-1)^{1/3}a^{1/3}}{b^{1/3}}+\frac{(-1)^{2/3}a^{1/3}}{b^{1/3}}}}$$

$$\left(\frac{1}{b^{1/3}}\left(-a^{1/3}+\left(-1\right)^{2/3}a^{1/3}\right)\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{\left(-1\right)^{2/3}a^{1/3}+b^{1/3}x}{\left(\left(-1\right)^{1/3}+\left(-1\right)^{2/3}\right)a^{1/3}}}\right],\right.\right.$$

$$\left.\left.\left.\frac{\left(-1\right)^{1/3}}{-1+\left(-1\right)^{1/3}}\right]+\frac{a^{1/3}\,\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(-1)^{2/3}a^{1/3}+b^{1/3}x}{\left((-1)^{1/3}+(-1)^{2/3}\right)a^{1/3}}}\right],\frac{(-1)^{1/3}}{-1+(-1)^{1/3}}\right]}{b^{1/3}}\right)\right)\Bigg/$$

$$\left(\sqrt{\frac{-\frac{(-1)^{1/3}a^{1/3}}{b^{1/3}}+x}{-\frac{(-1)^{1/3}a^{1/3}}{b^{1/3}}-\frac{(-1)^{2/3}a^{1/3}}{b^{1/3}}}}\sqrt{a+b\,x^3}\right)\Bigg)$$

Problem 495: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4\left(c+d\,x+e\,x^2+f\,x^3\right)\sqrt{a+b\,x^4}\,dx$$

Optimal (type 4, 418 leaves, 14 steps):

$$\frac{2\,a\,c\,x\,\sqrt{a+b\,x^4}}{21\,b}-\frac{a\,d\,x^2\,\sqrt{a+b\,x^4}}{16\,b}+\frac{2\,a\,e\,x^3\,\sqrt{a+b\,x^4}}{45\,b}-\frac{2\,a^2\,e\,x\,\sqrt{a+b\,x^4}}{15\,b^{3/2}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{63}\,x^5\,\left(9\,c+7\,e\,x^2\right)\,\sqrt{a+b\,x^4}+\frac{f\,x^4\,\left(a+b\,x^4\right)^{3/2}}{10\,b}-\frac{\left(8\,a\,f-15\,b\,d\,x^2\right)\,\left(a+b\,x^4\right)^{3/2}}{120\,b^2}-\frac{a^2\,d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{16\,b^{3/2}}+\frac{1}{15\,b^{7/4}\,\sqrt{a+b\,x^4}}2\,a^{9/4}\,e\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]-\left(a^{7/4}\,\left(5\,\sqrt{b}\,c+7\,\sqrt{a}\,e\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(105\,b^{7/4}\,\sqrt{a+b\,x^4}\right)$$

Result (type 4, 296 leaves):

$$\frac{1}{5040\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,b^2\,\sqrt{a+b\,x^4}}\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(-\left(a+b\,x^4\right)\,\left(336\,a^2\,f-2\,b^2\,x^5\,\left(360\,c+7\,x\,\left(45\,d+40\,e\,x+36\,f\,x^2\right)\right)-a\,b\,x\,\left(480\,c+7\,x\,\left(45\,d+8\,x\,\left(4\,e+3\,f\,x\right)\right)\right)\right)-315\,a^2\,\sqrt{b}\,d\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]\right)-672\,a^{5/2}\,\sqrt{b}\,e\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+96\,a^2\,\sqrt{b}\,\left(5\,i\,\sqrt{b}\,c+7\,\sqrt{a}\,e\right)\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 496: Result unnecessarily involves imaginary or complex numbers.

$$\int x^3\,\left(c+d\,x+e\,x^2+f\,x^3\right)\,\sqrt{a+b\,x^4}\,dx$$

Optimal (type 4, 394 leaves, 13 steps):

$$\frac{2 a d x \sqrt{a+b x^4}}{21 b}-\frac{a e x^2 \sqrt{a+b x^4}}{16 b}+\frac{2 a f x^3 \sqrt{a+b x^4}}{45 b}-\frac{2 a^2 f x \sqrt{a+b x^4}}{15 b^{3/2}\left(\sqrt{a}+\sqrt{b}\, x^2\right)}+$$

$$\frac{1}{63} x^5 \left(9 d+7 f x^2\right) \sqrt{a+b x^4}+\frac{\left(4 c+3 e x^2\right)\left(a+b x^4\right)^{3/2}}{24 b}-\frac{a^2 e \operatorname{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]}{16 b^{3/2}}+$$

$$\frac{1}{15 b^{7/4} \sqrt{a+b x^4}} 2 a^{9/4} f \left(\sqrt{a}+\sqrt{b}\, x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]-$$

$$\left(a^{7/4}\left(5 \sqrt{b}\, d+7 \sqrt{a}\, f\right)\left(\sqrt{a}+\sqrt{b}\, x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(105 b^{7/4} \sqrt{a+b x^4}\right)$$

Result (type 4, 275 leaves):

$$\frac{1}{5040 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, b^{3/2} \sqrt{a+b x^4}}$$

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \left(\sqrt{b}\left(a+b x^4\right)\left(10 b x^4 \left(84 c+x \left(72 d+7 x \left(9 e+8 f x\right)\right)\right)+a \left(840 c+\right.\right.\right.\right.$$

$$\left.\left.\left. x\left(480 d+7 x\left(45 e+32 f x\right)\right)\right)\right)-315 a^2 e \sqrt{a+b x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]\right)-$$

$$672 a^{5/2} f \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x\right], -1\right]+$$

$$\left.96 a^2 \left(5 i \sqrt{b}\, d+7 \sqrt{a}\, f\right) \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x\right], -1\right]\right)$$

Problem 497: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \left(c+d x+e x^2+f x^3\right) \sqrt{a+b x^4}\, dx$$

Optimal (type 4, 369 leaves, 12 steps):

$$\frac{2\,a\,e\,x\,\sqrt{a+b\,x^4}}{21\,b}-\frac{a\,f\,x^2\,\sqrt{a+b\,x^4}}{16\,b}+\frac{2\,a\,c\,x\,\sqrt{a+b\,x^4}}{5\,\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{35}\,x^3\,\left(7\,c+5\,e\,x^2\right)\,\sqrt{a+b\,x^4}+$$

$$\frac{\left(4\,d+3\,f\,x^2\right)\,\left(a+b\,x^4\right)^{3/2}}{24\,b}-\frac{a^2\,f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{16\,b^{3/2}}-\frac{1}{5\,b^{3/4}\,\sqrt{a+b\,x^4}}$$

$$2\,a^{5/4}\,c\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]+$$

$$\left(a^{5/4}\,\left(21\,\sqrt{b}\,c-5\,\sqrt{a}\,e\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$

$$\left(105\,b^{5/4}\,\sqrt{a+b\,x^4}\right)$$

Result (type 4, 280 leaves):

$$\frac{1}{1680\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,b^{3/2}\,\sqrt{a+b\,x^4}}\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\right.$$

$$\left(\sqrt{b}\,\left(a+b\,x^4\right)\,\left(5\,a\,\left(56\,d+x\,\left(32\,e+21\,f\,x\right)\right)+2\,b\,x^3\,\left(168\,c+5\,x\,\left(28\,d+3\,x\,\left(8\,e+7\,f\,x\right)\right)\right)\right)-\right.$$

$$\left.105\,a^2\,f\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]\right)+$$

$$672\,a^{3/2}\,b\,c\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]+$$

$$\left.32\,i\,a^{3/2}\,\sqrt{b}\,\left(21\,i\,\sqrt{b}\,c+5\,\sqrt{a}\,e\right)\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]\right)$$

Problem 498: Result unnecessarily involves imaginary or complex numbers.

$$\int x\,\left(c+d\,x+e\,x^2+f\,x^3\right)\,\sqrt{a+b\,x^4}\,dx$$

Optimal (type 4, 354 leaves, 12 steps):

$$\frac{2\,a\,f\,x\,\sqrt{a+b\,x^4}}{21\,b}+\frac{1}{4}\,c\,x^2\,\sqrt{a+b\,x^4}+\frac{2\,a\,d\,x\,\sqrt{a+b\,x^4}}{5\,\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+$$
$$\frac{1}{35}\,x^3\,\left(7\,d+5\,f\,x^2\right)\,\sqrt{a+b\,x^4}+\frac{e\,\left(a+b\,x^4\right)^{3/2}}{6\,b}+\frac{a\,c\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{4\,\sqrt{b}}-\frac{1}{5\,b^{3/4}\,\sqrt{a+b\,x^4}}$$
$$2\,a^{5/4}\,d\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]+$$
$$\left(a^{5/4}\,\left(21\,\sqrt{b}\,d-5\,\sqrt{a}\,f\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]\right)\bigg/$$
$$\left(105\,b^{5/4}\,\sqrt{a+b\,x^4}\right)$$

Result (type 4, 266 leaves):

$$\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(\left(a+b\,x^4\right)\,\left(10\,a\,\left(7\,e+4\,f\,x\right)+b\,x^2\,\left(105\,c+84\,d\,x+70\,e\,x^2+60\,f\,x^3\right)\right)+\right.\right.$$
$$\left.105\,a\,\sqrt{b}\,c\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]\right)+$$
$$168\,a^{3/2}\,\sqrt{b}\,d\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]+$$
$$\left.8\,i\,a^{3/2}\,\left(21\,i\,\sqrt{b}\,d+5\,\sqrt{a}\,f\right)\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]\right)\bigg/$$
$$\left(420\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,b\,\sqrt{a+b\,x^4}\right)$$

Problem 499: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(c+d\,x+e\,x^2+f\,x^3\right)\,\sqrt{a+b\,x^4}\,dx$$

Optimal (type 4, 331 leaves, 11 steps):

$$\frac{1}{4} d x^2 \sqrt{a+b x^4}+\frac{2 a e x \sqrt{a+b x^4}}{5 \sqrt{b} \left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{1}{15} x \left(5 c+3 e x^2\right) \sqrt{a+b x^4}+$$

$$\frac{f \left(a+b x^4\right)^{3/2}}{6 b}+\frac{a d \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{4 \sqrt{b}}-\frac{1}{5 b^{3/4} \sqrt{a+b x^4}}$$

$$2 a^{5/4} e \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]+\frac{1}{15 b^{3/4} \sqrt{a+b x^4}}$$

$$a^{3/4} \left(5 \sqrt{b} c+3 \sqrt{a} e\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 257 leaves) :

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \left(\left(a+b x^4\right) \left(10 a f+b x \left(20 c+x \left(15 d+2 x \left(6 e+5 f x\right)\right)\right)\right)+\right.\right.$$

$$\left.15 a \sqrt{b} d \sqrt{a+b x^4} \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]\right)+$$

$$24 a^{3/2} \sqrt{b} e \sqrt{1+\frac{b x^4}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]-$$

$$\left.8 a \sqrt{b} \left(5 i \sqrt{b} c+3 \sqrt{a} e\right) \sqrt{1+\frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right) \Big/$$

$$\left(60 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b \sqrt{a+b x^4}\right)$$

Problem 500: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(c+d x+e x^2+f x^3\right) \sqrt{a+b x^4}}{x} \, dx$$

Optimal (type 4, 345 leaves, 14 steps) :

$$\frac{2\,a\,f\,x\,\sqrt{a+b\,x^4}}{5\,\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{4}\,\left(2\,c+e\,x^2\right)\,\sqrt{a+b\,x^4}+\frac{1}{15}\,x\,\left(5\,d+3\,f\,x^2\right)\,\sqrt{a+b\,x^4}+$$

$$\frac{a\,e\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{4\,\sqrt{b}}-\frac{1}{2}\,\sqrt{a}\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]-\frac{1}{5\,b^{3/4}\,\sqrt{a+b\,x^4}}$$

$$2\,a^{5/4}\,f\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]+\frac{1}{15\,b^{3/4}\,\sqrt{a+b\,x^4}}$$

$$a^{3/4}\,\left(5\,\sqrt{b}\,d+3\,\sqrt{a}\,f\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]$$

Result (type 4, 280 leaves):

$$\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(15\,a\,e\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]+\sqrt{b}\right.\right.$$

$$\left.\left.\left(\left(a+b\,x^4\right)\,\left(30\,c+x\,\left(20\,d+3\,x\,\left(5\,e+4\,f\,x\right)\right)\right)-30\,\sqrt{a}\,c\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)\right)+$$

$$24\,a^{3/2}\,f\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]-$$

$$\left.8\,a\,\left(5\,i\,\sqrt{b}\,d+3\,\sqrt{a}\,f\right)\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]\right)\Big/$$

$$\left(60\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\sqrt{b}\,\sqrt{a+b\,x^4}\right)$$

Problem 501: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\,\sqrt{a+b\,x^4}}{x^2}\,dx$$

Optimal (type 4, 341 leaves, 14 steps):

$$\frac{2\sqrt{b}\,c\,x\sqrt{a+b\,x^4}}{\sqrt{a}+\sqrt{b}\,x^2}-\frac{\left(3\,c-e\,x^2\right)\sqrt{a+b\,x^4}}{3\,x}+\frac{1}{4}\left(2\,d+f\,x^2\right)\sqrt{a+b\,x^4}+$$

$$\frac{a\,f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{4\sqrt{b}}-\frac{1}{2}\sqrt{a}\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]-\frac{1}{\sqrt{a+b\,x^4}}$$

$$2\,a^{1/4}\,b^{1/4}\,c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]+\frac{1}{3\,b^{1/4}\sqrt{a+b\,x^4}}$$

$$a^{1/4}\left(3\sqrt{b}\,c+\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 355 leaves):

$$\left(\frac{d}{2}-\frac{c}{x}+\frac{e\,x}{3}+\frac{f\,x^2}{4}\right)\sqrt{a+b\,x^4}+\frac{1}{6}\left(\frac{3\,a\,f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{2\sqrt{b}}-3\sqrt{a}\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]+\right.$$

$$\left(12\sqrt{a}\,\sqrt{b}\,c\sqrt{1-\frac{\mathrm{i}\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\mathrm{i}\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-\right.\right.$$

$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)\right)\Big/\left(\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\,x^4}\right)-$$

$$\left(4\,\mathrm{i}\,a\,e\sqrt{1-\frac{\mathrm{i}\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\mathrm{i}\sqrt{b}\,x^2}{\sqrt{a}}}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)\Big/$$

$$\left.\left(\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\sqrt{a+b\,x^4}\right)\right)$$

Problem 502: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\sqrt{a+b\,x^4}}{x^3}\,dx$$

Optimal (type 4, 342 leaves, 14 steps):

$$\frac{2\sqrt{b}\,d\,x\sqrt{a+b\,x^4}}{\sqrt{a}+\sqrt{b}\,x^2}-\frac{\left(c-e\,x^2\right)\sqrt{a+b\,x^4}}{2\,x^2}-\frac{\left(3\,d-f\,x^2\right)\sqrt{a+b\,x^4}}{3\,x}+$$

$$\frac{1}{2}\sqrt{b}\,c\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]-\frac{1}{2}\sqrt{a}\,e\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]-\frac{1}{\sqrt{a+b\,x^4}}$$

$$2\,a^{1/4}\,b^{1/4}\,d\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]+\frac{1}{3\,b^{1/4}\sqrt{a+b\,x^4}}$$

$$a^{1/4}\left(3\sqrt{b}\,d+\sqrt{a}\,f\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 296 leaves):

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+b\,x^4\right)\left(-3\,c+x\left(-6\,d+3\,e\,x+2\,f\,x^2\right)\right)+\right.\right.$$

$$\left.3\sqrt{b}\,c\,x^2\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]-3\sqrt{a}\,e\,x^2\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+$$

$$12\sqrt{a}\sqrt{b}\,d\,x^2\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-$$

$$\left.4\,i\sqrt{a}\left(-3\,i\sqrt{b}\,d+\sqrt{a}\,f\right)x^2\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)\Big/$$

$$\left(6\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^2\sqrt{a+b\,x^4}\right)$$

Problem 503: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\sqrt{a+b\,x^4}}{x^4}\,dx$$

Optimal (type 4, 357 leaves, 15 steps):

$$-\frac{2 e \sqrt{a+b x^4}}{x}+\frac{2 \sqrt{b}\, e x \sqrt{a+b x^4}}{\sqrt{a}+\sqrt{b}\, x^2}-\frac{\left(c-3 e x^2\right) \sqrt{a+b x^4}}{3 x^3}-\frac{\left(d-f x^2\right) \sqrt{a+b x^4}}{2 x^2}+$$
$$\frac{1}{2} \sqrt{b}\, d \,\text{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]-\frac{1}{2} \sqrt{a}\, f \,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]-\frac{1}{\sqrt{a+b x^4}}$$
$$2 a^{1/4} b^{1/4} e \left(\sqrt{a}+\sqrt{b}\, x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}} \,\text{EllipticE}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]+\frac{1}{3 a^{1/4} \sqrt{a+b x^4}}$$
$$b^{1/4} \left(\sqrt{b}\, c+3 \sqrt{a}\, e\right) \left(\sqrt{a}+\sqrt{b}\, x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}} \,\text{EllipticF}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 295 leaves):

$$\left(-\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \left(\left(a+b x^4\right) \left(2 c+3 x \left(d+2 e x-f x^2\right)\right)-\right.\right.$$
$$\left.3 \sqrt{b}\, d x^3 \sqrt{a+b x^4} \,\text{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]+3 \sqrt{a}\, f x^3 \sqrt{a+b x^4} \,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right)+$$
$$12 \sqrt{a} \sqrt{b}\, e x^3 \sqrt{1+\frac{b x^4}{a}} \,\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]-$$
$$\left.4 \sqrt{b} \left(i \sqrt{b}\, c+3 \sqrt{a}\, e\right) x^3 \sqrt{1+\frac{b x^4}{a}} \,\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]\right) \Big/$$
$$\left(6 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x^3 \sqrt{a+b x^4}\right)$$

Problem 504: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(c+d x+e x^2+f x^3\right) \sqrt{a+b x^4}}{x^5} \, dx$$

Optimal (type 4, 329 leaves, 13 steps):

$$-\frac{1}{12}\left(\frac{3\,c}{x^4}+\frac{4\,d}{x^3}+\frac{6\,e}{x^2}+\frac{12\,f}{x}\right)\sqrt{a+b\,x^4}+\frac{2\,\sqrt{b}\,f\,x\,\sqrt{a+b\,x^4}}{\sqrt{a}+\sqrt{b}\,x^2}+$$
$$\frac{1}{2}\sqrt{b}\,e\,\text{ArcTanh}\Big[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\Big]-\frac{b\,c\,\text{ArcTanh}\Big[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\Big]}{4\,\sqrt{a}}-\frac{1}{\sqrt{a+b\,x^4}}$$
$$2\,a^{1/4}\,b^{1/4}\,f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\Big[2\,\text{ArcTan}\Big[\frac{b^{1/4}\,x}{a^{1/4}}\Big],\frac{1}{2}\Big]+\frac{1}{3\,a^{1/4}\,\sqrt{a+b\,x^4}}$$
$$b^{1/4}\left(\sqrt{b}\,d+3\,\sqrt{a}\,f\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\Big[2\,\text{ArcTan}\Big[\frac{b^{1/4}\,x}{a^{1/4}}\Big],\frac{1}{2}\Big]$$

Result (type 4, 267 leaves) :

$$\frac{1}{12}$$
$$\left(-\frac{\sqrt{a+b\,x^4}\,\left(3\,c+4\,d\,x+6\,x^2\,(e+2\,f\,x)\right)}{x^4}+6\,\sqrt{b}\,e\,\text{ArcTanh}\Big[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\Big]-\frac{3\,b\,c\,\text{ArcTanh}\Big[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\Big]}{\sqrt{a}}-\right.$$
$$\frac{24\,i\,a\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,f\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\Big],-1\Big]}{\sqrt{a+b\,x^4}}-\frac{1}{\sqrt{a+b\,x^4}}$$
$$\left.8\,\sqrt{a}\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(\sqrt{b}\,d-3\,i\,\sqrt{a}\,f\right)\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\Big],-1\Big]\right)$$

Problem 505: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\sqrt{a+b\,x^4}}{x^6}\,dx$$

Optimal (type 4, 360 leaves, 14 steps) :

$$-\frac{1}{60}\left(\frac{12\,c}{x^5}+\frac{15\,d}{x^4}+\frac{20\,e}{x^3}+\frac{30\,f}{x^2}\right)\sqrt{a+b\,x^4}-\frac{2\,b\,c\,\sqrt{a+b\,x^4}}{5\,a\,x}+\frac{2\,b^{3/2}\,c\,x\,\sqrt{a+b\,x^4}}{5\,a\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+$$

$$\frac{1}{2}\sqrt{b}\,f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]-\frac{b\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{4\,\sqrt{a}}-\frac{1}{5\,a^{3/4}\,\sqrt{a+b\,x^4}}$$

$$2\,b^{5/4}\,c\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]+\frac{1}{15\,a^{3/4}\,\sqrt{a+b\,x^4}}$$

$$b^{3/4}\left(3\,\sqrt{b}\,c+5\,\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 314 leaves):

$$\frac{1}{60\,a\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\;x^5\,\sqrt{a+b\,x^4}}$$

$$\left(-\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\left(\left(a+b\,x^4\right)\left(12\,a\,c+24\,b\,c\,x^4+5\,a\,x\left(3\,d+4\,e\,x+6\,f\,x^2\right)\right)-30\,a\,\sqrt{b}\,f\,x^5\right.\right.$$

$$\left.\sqrt{a+b\,x^4}\;\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]+15\,\sqrt{a}\,b\,d\,x^5\,\sqrt{a+b\,x^4}\;\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+$$

$$24\,\sqrt{a}\,b^{3/2}\,c\,x^5\,\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-$$

$$\left.8\,i\,\sqrt{a}\,b\left(-3\,i\,\sqrt{b}\,c+5\,\sqrt{a}\,e\right)x^5\,\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 506: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\sqrt{a+b\,x^4}}{x^7}\,dx$$

Optimal (type 4, 352 leaves, 12 steps):

$$-\frac{1}{60}\left(\frac{10\,c}{x^6}+\frac{12\,d}{x^5}+\frac{15\,e}{x^4}+\frac{20\,f}{x^3}\right)\sqrt{a+b\,x^4}-\frac{b\,c\,\sqrt{a+b\,x^4}}{6\,a\,x^2}-$$

$$\frac{2\,b\,d\,\sqrt{a+b\,x^4}}{5\,a\,x}+\frac{2\,b^{3/2}\,d\,x\,\sqrt{a+b\,x^4}}{5\,a\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{b\,e\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{4\,\sqrt{a}}-\frac{1}{5\,a^{3/4}\,\sqrt{a+b\,x^4}}$$

$$2\,b^{5/4}\,d\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]+\frac{1}{15\,a^{3/4}\,\sqrt{a+b\,x^4}}$$

$$b^{3/4}\,\left(3\,\sqrt{b}\,d+5\,\sqrt{a}\,f\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 277 leaves):

$$\left(-\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(\left(a+b\,x^4\right)\,\left(10\,a\,c+2\,b\,x^4\,\left(5\,c+12\,d\,x\right)+a\,x\,\left(12\,d+5\,x\,\left(3\,e+4\,f\,x\right)\right)\right)+\right.\right.$$

$$\left.15\,\sqrt{a}\,b\,e\,x^6\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+$$

$$24\,\sqrt{a}\,b^{3/2}\,d\,x^6\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-$$

$$\left.8\,i\,\sqrt{a}\,b\,\left(-3\,i\,\sqrt{b}\,d+5\,\sqrt{a}\,f\right)\,x^6\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)\Big/$$

$$\left(60\,a\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^6\,\sqrt{a+b\,x^4}\right)$$

Problem 507: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\,\sqrt{a+b\,x^4}}{x^8}\,dx$$

Optimal (type 4, 375 leaves, 13 steps):

$$-\frac{1}{420}\left(\frac{60\,c}{x^7}+\frac{70\,d}{x^6}+\frac{84\,e}{x^5}+\frac{105\,f}{x^4}\right)\sqrt{a+b\,x^4}-\frac{2\,b\,c\,\sqrt{a+b\,x^4}}{21\,a\,x^3}-\frac{b\,d\,\sqrt{a+b\,x^4}}{6\,a\,x^2}-$$
$$\frac{2\,b\,e\,\sqrt{a+b\,x^4}}{5\,a\,x}+\frac{2\,b^{3/2}\,e\,x\,\sqrt{a+b\,x^4}}{5\,a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{b\,f\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{4\,\sqrt{a}}-\frac{1}{5\,a^{3/4}\,\sqrt{a+b\,x^4}}$$
$$2\,b^{5/4}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]-$$
$$\left(b^{5/4}\left(5\,\sqrt{b}\,c-21\,\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(105\,a^{5/4}\,\sqrt{a+b\,x^4}\right)$$

Result (type 4, 283 leaves) :

$$\frac{1}{420\,a\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^7\,\sqrt{a+b\,x^4}}$$
$$\left(-\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\left(\left(a+b\,x^4\right)\left(2\,b\,x^4\left(20\,c+7\,x\left(5\,d+12\,e\,x\right)\right)+a\left(60\,c+7\,x\left(10\,d+3\,x\left(4\,e+5\,f\,x\right)\right)\right)\right)+\right.\right.$$
$$\left.105\,\sqrt{a}\,b\,f\,x^7\,\sqrt{a+b\,x^4}\;\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+$$
$$168\,\sqrt{a}\,b^{3/2}\,e\,x^7\,\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-$$
$$\left.8\,b^{3/2}\left(-5\,i\,\sqrt{b}\,c+21\,\sqrt{a}\,e\right)x^7\,\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 508: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\sqrt{a+b\,x^4}}{x^9}\,dx$$

Optimal (type 4, 400 leaves, 14 steps) :

$$-\frac{1}{840}\left(\frac{105\,c}{x^8}+\frac{120\,d}{x^7}+\frac{140\,e}{x^6}+\frac{168\,f}{x^5}\right)\sqrt{a+b\,x^4}-\frac{b\,c\,\sqrt{a+b\,x^4}}{16\,a\,x^4}-\frac{2\,b\,d\,\sqrt{a+b\,x^4}}{21\,a\,x^3}-\frac{b\,e\,\sqrt{a+b\,x^4}}{6\,a\,x^2}-\frac{2\,b\,f\,\sqrt{a+b\,x^4}}{5\,a\,x}+\frac{2\,b^{3/2}\,f\,x\,\sqrt{a+b\,x^4}}{5\,a\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{b^2\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{16\,a^{3/2}}-\frac{1}{5\,a^{3/4}\,\sqrt{a+b\,x^4}}2\,b^{5/4}\,f\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]-\left(b^{5/4}\,\left(5\,\sqrt{b}\,d-21\,\sqrt{a}\,f\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(105\,a^{5/4}\,\sqrt{a+b\,x^4}\right)$$

Result (type 4, 293 leaves) :

$$\frac{1}{1680\,a^{3/2}\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^8\,\sqrt{a+b\,x^4}}\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\left(-\sqrt{a}\,\left(a+b\,x^4\right)\left(b\,x^4\,\left(105\,c+8\,x\,\left(20\,d+35\,e\,x+84\,f\,x^2\right)\right)+a\,\left(210\,c+8\,x\,\left(30\,d+7\,x\,\left(5\,e+6\,f\,x\right)\right)\right)\right)+105\,b^2\,c\,x^8\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+672\,a\,b^{3/2}\,f\,x^8\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-32\,\sqrt{a}\,b^{3/2}\left(-5\,i\,\sqrt{b}\,d+21\,\sqrt{a}\,f\right)x^8\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 509: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\sqrt{a+b\,x^4}}{x^{10}}\,dx$$

Optimal (type 4, 425 leaves, 15 steps) :

$$-\frac{1}{504}\left(\frac{56\,c}{x^9}+\frac{63\,d}{x^8}+\frac{72\,e}{x^7}+\frac{84\,f}{x^6}\right)\sqrt{a+b\,x^4}-\frac{2\,b\,c\,\sqrt{a+b\,x^4}}{45\,a\,x^5}-\frac{b\,d\,\sqrt{a+b\,x^4}}{16\,a\,x^4}-\frac{2\,b\,e\,\sqrt{a+b\,x^4}}{21\,a\,x^3}-$$

$$\frac{b\,f\,\sqrt{a+b\,x^4}}{6\,a\,x^2}+\frac{2\,b^2\,c\,\sqrt{a+b\,x^4}}{15\,a^2\,x}-\frac{2\,b^{5/2}\,c\,x\,\sqrt{a+b\,x^4}}{15\,a^2\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{b^2\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{16\,a^{3/2}}+$$

$$\frac{1}{15\,a^{7/4}\,\sqrt{a+b\,x^4}}2\,b^{9/4}\,c\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]-$$

$$\left(b^{7/4}\,\left(7\,\sqrt{b}\,c+5\,\sqrt{a}\,e\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(105\,a^{7/4}\,\sqrt{a+b\,x^4}\right)$$

Result (type 4, 305 leaves):

$$\frac{1}{5040\,a^2\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\;x^9\,\sqrt{a+b\,x^4}}$$

$$\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\left(-\left(a+b\,x^4\right)\,\left(-672\,b^2\,c\,x^8+10\,a^2\,\left(56\,c+63\,d\,x+72\,e\,x^2+84\,f\,x^3\right)+a\,b\,x^4\,\left(224\,c+\right.\right.\right.\right.$$

$$\left.\left.\left.15\,x\,\left(21\,d+8\,x\,\left(4\,e+7\,f\,x\right)\right)\right)\right)+315\,\sqrt{a}\,b^2\,d\,x^9\,\sqrt{a+b\,x^4}\;\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)-$$

$$672\,\sqrt{a}\,b^{5/2}\,c\,x^9\,\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+$$

$$\left.96\,\sqrt{a}\,b^2\,\left(7\,\sqrt{b}\,c+5\,i\,\sqrt{a}\,e\right)\,x^9\,\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 510: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4\,\left(c+d\,x+e\,x^2+f\,x^3\right)\,\left(a+b\,x^4\right)^{3/2}\,dx$$

Optimal (type 4, 476 leaves, 16 steps):

$$\frac{4 a^2 c x \sqrt{a+b x^4}}{77 b}-\frac{a^2 d x^2 \sqrt{a+b x^4}}{32 b}+\frac{4 a^2 e x^3 \sqrt{a+b x^4}}{195 b}-\frac{4 a^3 e x \sqrt{a+b x^4}}{65 b^{3/2}\left(\sqrt{a}+\sqrt{b} x^2\right)}+$$

$$\frac{2 a x^5\left(117 c+77 e x^2\right) \sqrt{a+b x^4}}{3003}-\frac{a d x^2\left(a+b x^4\right)^{3/2}}{48 b}+\frac{1}{143} x^5\left(13 c+11 e x^2\right)\left(a+b x^4\right)^{3/2}+$$

$$\frac{f x^4\left(a+b x^4\right)^{5/2}}{14 b}-\frac{\left(12 a f-35 b d x^2\right)\left(a+b x^4\right)^{5/2}}{420 b^2}-\frac{a^3 d \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{32 b^{3/2}}+\frac{1}{65 b^{7/4} \sqrt{a+b x^4}}$$

$$4 a^{13/4} e\left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]-$$

$$\left(2 a^{11/4}\left(65 \sqrt{b} c+77 \sqrt{a} e\right)\left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}}\right.$$

$$\left.\operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(5005 b^{7/4} \sqrt{a+b x^4}\right)$$

Result (type 4, 327 leaves):

$$\frac{1}{480480 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b^2 \sqrt{a+b x^4}}$$

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\left(-\left(a+b x^4\right)\left(13728 a^3 f-40 b^3 x^9\left(1092 c+11 x\left(91 d+84 e x+78 f x^2\right)\right)-\right.\right.\right.$$

$$a^2 b x\left(24960 c+11 x\left(1365 d+896 e x+624 f x^2\right)\right)-$$

$$\left.\left.2 a b^2 x^5\left(40560 c+11 x\left(3185 d+2800 e x+2496 f x^2\right)\right)\right)-$$

$$\left.15015 a^3 \sqrt{b} d \sqrt{a+b x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]\right)-$$

$$29568 a^{7/2} \sqrt{b} e \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]+$$

$$384 a^3 \sqrt{b}\left(65 i \sqrt{b} c+77 \sqrt{a} e\right) \sqrt{1+\frac{b x^4}{a}}$$

$$\left.\operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right)$$

Problem 511: Result unnecessarily involves imaginary or complex numbers.

$$\int x^3 \left(c + d\,x + e\,x^2 + f\,x^3\right) \left(a + b\,x^4\right)^{3/2} dx$$

Optimal (type 4, 452 leaves, 15 steps):

$$\frac{4\,a^2\,d\,x\,\sqrt{a+b\,x^4}}{77\,b} - \frac{a^2\,e\,x^2\,\sqrt{a+b\,x^4}}{32\,b} + \frac{4\,a^2\,f\,x^3\,\sqrt{a+b\,x^4}}{195\,b} - \frac{4\,a^3\,f\,x\,\sqrt{a+b\,x^4}}{65\,b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x^2\right)} + \frac{2\,a\,x^5\left(117\,d+77\,f\,x^2\right)\sqrt{a+b\,x^4}}{3003} - \frac{a\,e\,x^2\left(a+b\,x^4\right)^{3/2}}{48\,b} + \frac{1}{143}\,x^5\left(13\,d+11\,f\,x^2\right)\left(a+b\,x^4\right)^{3/2} + \frac{\left(6\,c+5\,e\,x^2\right)\left(a+b\,x^4\right)^{5/2}}{60\,b} - \frac{a^3\,e\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{32\,b^{3/2}} + \frac{1}{65\,b^{7/4}\,\sqrt{a+b\,x^4}}\,4\,a^{13/4}\,f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right] - \left(2\,a^{11/4}\left(65\,\sqrt{b}\,d+77\,\sqrt{a}\,f\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(5005\,b^{7/4}\,\sqrt{a+b\,x^4}\right)$$

Result (type 4, 306 leaves):

$$\frac{1}{480480\,b^2\,\sqrt{a+b\,x^4}}\left(b\left(a+b\,x^4\right)\left(56\,b^2\,x^8\left(858\,c+780\,d\,x+715\,e\,x^2+660\,f\,x^3\right)+2\,a\,b\,x^4\left(48048\,c+5\,x\left(8112\,d+77\,x\left(91\,e+80\,f\,x\right)\right)\right)+a^2\left(48048\,c+x\left(24960\,d+77\,x\left(195\,e+128\,f\,x\right)\right)\right)\right) - 15015\,a^3\,\sqrt{b}\,e\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right] + 29568\,i\,a^4\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,f\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right] + 384\,a^{7/2}\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\left(65\,\sqrt{b}\,d-77\,i\,\sqrt{a}\,f\right)\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 512: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \left(c+d x+e x^2+f x^3\right) \left(a+b x^4\right)^{3/2} \, dx$$

Optimal (type 4, 427 leaves, 14 steps):

$$\frac{4 a^2 e x \sqrt{a+b x^4}}{77 b}-\frac{a^2 f x^2 \sqrt{a+b x^4}}{32 b}+\frac{4 a^2 c x \sqrt{a+b x^4}}{15 \sqrt{b} \left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{2 a x^3 \left(77 c+45 e x^2\right) \sqrt{a+b x^4}}{1155}-\frac{a f x^2 \left(a+b x^4\right)^{3/2}}{48 b}+\frac{1}{99} x^3 \left(11 c+9 e x^2\right) \left(a+b x^4\right)^{3/2}+\frac{\left(6 d+5 f x^2\right) \left(a+b x^4\right)^{5/2}}{60 b}-\frac{a^3 f \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{32 b^{3/2}}-\frac{1}{15 b^{3/4} \sqrt{a+b x^4}} 4 a^{9/4} c \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]+\left(2 a^{9/4} \left(77 \sqrt{b} c-15 \sqrt{a} e\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(1155 b^{5/4} \sqrt{a+b x^4}\right)$$

Result (type 4, 325 leaves):

$$\frac{1}{110880 b}\sqrt{a+b x^4} \left(9 a^2 \left(1232 d+5 x \left(128 e+77 f x\right)\right)+56 b^2 x^7 \left(220 c+3 x \left(66 d+60 e x+55 f x^2\right)\right)+2 a b x^3 \left(13552 c+3 x \left(3696 d+5 x \left(624 e+539 f x\right)\right)\right)\right)-\frac{a^3 f \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{32 b^{3/2}}+\left(4 i a^2 c \sqrt{1+\frac{b x^4}{a}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]-\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right)\right)\Big/\left(15 \left(\frac{i \sqrt{b}}{\sqrt{a}}\right)^{3/2} \sqrt{a+b x^4}\right)+\frac{4 i a^3 e \sqrt{1+\frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]}{77 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b \sqrt{a+b x^4}}$$

Problem 513: Result unnecessarily involves imaginary or complex numbers.

$$\int x\left(c+d x+e x^2+f x^3\right)\left(a+b x^4\right)^{3/2}\,dx$$

Optimal (type 4, 409 leaves, 14 steps):

$$\frac{4 a^2 f x \sqrt{a+b x^4}}{77 b}+\frac{3}{16} a c x^2 \sqrt{a+b x^4}+\frac{4 a^2 d x \sqrt{a+b x^4}}{15 \sqrt{b}\left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{2 a x^3\left(77 d+45 f x^2\right) \sqrt{a+b x^4}}{1155}+$$

$$\frac{1}{8} c x^2\left(a+b x^4\right)^{3/2}+\frac{1}{99} x^3\left(11 d+9 f x^2\right)\left(a+b x^4\right)^{3/2}+\frac{e\left(a+b x^4\right)^{5/2}}{10 b}+\frac{3 a^2 c \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{16 \sqrt{b}}-$$

$$\frac{1}{15 b^{3/4} \sqrt{a+b x^4}} 4 a^{9/4} d\left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]+$$

$$\left(2 a^{9/4}\left(77 \sqrt{b} d-15 \sqrt{a} f\right)\left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(1155 b^{5/4} \sqrt{a+b x^4}\right)$$

Result (type 4, 302 leaves):

$$\frac{1}{55440 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, b \sqrt{a+b x^4}}$$

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\left(\left(a+b x^4\right)\left(72 a^2(77 e+40 f x)+14 b^2 x^6\left(495 c+4 x\left(110 d+99 e x+90 f x^2\right)\right)+\right.\right.\right.$$

$$\left.\left. a b x^2(17325 c+16 x(847 d+9 x(77 e+65 f x)))\right)+\right.$$

$$\left. 10395 a^2 \sqrt{b}\, c \sqrt{a+b x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]\right)+$$

$$14784 a^{5/2} \sqrt{b}\, d \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]+$$

$$\left.192\, i\, a^{5/2}\left(77\, i \sqrt{b}\, d+15 \sqrt{a} f\right) \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]\right)$$

Problem 514: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(c+d x+e x^2+f x^3\right)\left(a+b x^4\right)^{3/2}\,dx$$

Optimal (type 4, 382 leaves, 13 steps):

$$\frac{3}{16} a d x^2 \sqrt{a+b x^4}+\frac{4 a^2 e x \sqrt{a+b x^4}}{15 \sqrt{b} \left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{2}{105} a x \left(15 c+7 e x^2\right) \sqrt{a+b x^4}+\frac{1}{8} d x^2 \left(a+b x^4\right)^{3/2}+$$
$$\frac{1}{63} x \left(9 c+7 e x^2\right) \left(a+b x^4\right)^{3/2}+\frac{f \left(a+b x^4\right)^{5/2}}{10 b}+\frac{3 a^2 d \,\text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{16 \sqrt{b}}-\frac{1}{15 b^{3/4} \sqrt{a+b x^4}}$$
$$4 a^{9/4} e \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \,\text{EllipticE}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]+$$
$$\left(2 a^{7/4} \left(15 \sqrt{b} c+7 \sqrt{a} e\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \,\text{EllipticF}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$
$$\left(105 b^{3/4} \sqrt{a+b x^4}\right)$$

Result (type 4, 294 leaves):

$$\frac{1}{5040 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b \sqrt{a+b x^4}}$$
$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \left(\left(a+b x^4\right) \left(504 a^2 f+2 b^2 x^5 \left(360 c+7 x \left(45 d+40 e x+36 f x^2\right)\right)+a b x \left(2160 c+\right.\right.\right.\right.$$
$$\left.\left.\left. 7 x \left(225 d+16 x \left(11 e+9 f x\right)\right)\right)\right)+945 a^2 \sqrt{b} d \sqrt{a+b x^4} \,\text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]\right)+$$
$$1344 a^{5/2} \sqrt{b} e \sqrt{1+\frac{b x^4}{a}} \,\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]-$$
$$\left. 192 a^2 \sqrt{b} \left(15 i \sqrt{b} c+7 \sqrt{a} e\right) \sqrt{1+\frac{b x^4}{a}} \,\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]\right)$$

Problem 515: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(c+d x+e x^2+f x^3\right) \left(a+b x^4\right)^{3/2}}{x} \, dx$$

Optimal (type 4, 403 leaves, 16 steps):

$$\frac{4 a^2 f x \sqrt{a+b x^4}}{15 \sqrt{b} \left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{1}{16} a \left(8 c+3 e x^2\right) \sqrt{a+b x^4}+\frac{2}{105} a x \left(15 d+7 f x^2\right) \sqrt{a+b x^4}+\frac{1}{24} \left(4 c+3 e x^2\right) \left(a+b x^4\right)^{3/2}+\frac{1}{63} x \left(9 d+7 f x^2\right) \left(a+b x^4\right)^{3/2}+\frac{3 a^2 e \,\text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{16 \sqrt{b}}-\frac{1}{2} a^{3/2} c \,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]-\frac{1}{15 b^{3/4} \sqrt{a+b x^4}} 4 a^{9/4} f \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \,\text{EllipticE}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]+\left(2 a^{7/4} \left(15 \sqrt{b} d+7 \sqrt{a} f\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \,\text{EllipticF}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(105 b^{3/4} \sqrt{a+b x^4}\right)$$

Result (type 4, 319 leaves):

$$\frac{1}{5040}\sqrt{a+b x^4} \left(10 b x^4 \left(84 c+x \left(72 d+7 x \left(9 e+8 f x\right)\right)\right)+a \left(3360 c+x \left(2160 d+7 x \left(225 e+176 f x\right)\right)\right)\right)+\frac{3 a^2 e \,\text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{16 \sqrt{b}}-\frac{1}{2} a^{3/2} c \,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]+\left(4 i a^2 f \sqrt{1+\frac{b x^4}{a}} \left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]-\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right)\right)\Big/\left(15 \left(\frac{i \sqrt{b}}{\sqrt{a}}\right)^{3/2} \sqrt{a+b x^4}\right)-\frac{4 i a^2 d \sqrt{1+\frac{b x^4}{a}} \,\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]}{7 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \sqrt{a+b x^4}}$$

Problem 516: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(c+d x+e x^2+f x^3\right) \left(a+b x^4\right)^{3/2}}{x^2} \, dx$$

Optimal (type 4, 404 leaves, 16 steps):

$$\frac{12\,a\,\sqrt{b}\,c\,x\,\sqrt{a+b\,x^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{2}{35}\,x\left(5\,a\,e+21\,b\,c\,x^2\right)\sqrt{a+b\,x^4}+$$
$$\frac{1}{16}\,a\left(8\,d+3\,f\,x^2\right)\sqrt{a+b\,x^4}-\frac{\left(7\,c-e\,x^2\right)\left(a+b\,x^4\right)^{3/2}}{7\,x}+\frac{1}{24}\left(4\,d+3\,f\,x^2\right)\left(a+b\,x^4\right)^{3/2}+$$
$$\frac{3\,a^2\,f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{16\,\sqrt{b}}-\frac{1}{2}\,a^{3/2}\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]-\frac{1}{5\,\sqrt{a+b\,x^4}}$$
$$12\,a^{5/4}\,b^{1/4}\,c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]+\frac{1}{35\,b^{1/4}\,\sqrt{a+b\,x^4}}$$
$$2\,a^{5/4}\left(21\,\sqrt{b}\,c+5\,\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 328 leaves):

$$\sqrt{a+b\,x^4}\left(a\left(\frac{2\,d}{3}-\frac{c}{x}+\frac{3\,e\,x}{7}+\frac{5\,f\,x^2}{16}\right)+b\left(\frac{c\,x^3}{5}+\frac{d\,x^4}{6}+\frac{e\,x^5}{7}+\frac{f\,x^6}{8}\right)\right)+$$
$$\frac{3\,a^2\,f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{16\,\sqrt{b}}-\frac{1}{2}\,a^{3/2}\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]+\left(12\,i\,a\,b\,c\,\sqrt{1+\frac{b\,x^4}{a}}\right.$$
$$\left.\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)\right)\Big/$$
$$\left(5\left(\frac{i\,\sqrt{b}}{\sqrt{a}}\right)^{3/2}\sqrt{a+b\,x^4}\right)-\frac{4\,i\,a^2\,e\,\sqrt{1+\frac{b\,x^4}{a}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]}{7\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\sqrt{a+b\,x^4}}$$

Problem 517: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\left(a+b\,x^4\right)^{3/2}}{x^3}\,dx$$

Optimal (type 4, 406 leaves, 16 steps):

$$\frac{12\,a\,\sqrt{b}\,d\,x\,\sqrt{a+b\,x^4}}{5\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{1}{4}\,\left(2\,a\,e+3\,b\,c\,x^2\right)\,\sqrt{a+b\,x^4}+$$
$$\frac{2}{35}\,x\,\left(5\,a\,f+21\,b\,d\,x^2\right)\,\sqrt{a+b\,x^4}-\frac{\left(3\,c-e\,x^2\right)\,\left(a+b\,x^4\right)^{3/2}}{6\,x^2}-\frac{\left(7\,d-f\,x^2\right)\,\left(a+b\,x^4\right)^{3/2}}{7\,x}+$$
$$\frac{3}{4}\,a\,\sqrt{b}\,c\,\text{ArcTanh}\Big[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\Big]-\frac{1}{2}\,a^{3/2}\,e\,\text{ArcTanh}\Big[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\Big]-\frac{1}{5\,\sqrt{a+b\,x^4}}$$
$$12\,a^{5/4}\,b^{1/4}\,d\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\Big[2\,\text{ArcTan}\Big[\frac{b^{1/4}\,x}{a^{1/4}}\Big],\,\frac{1}{2}\Big]+\frac{1}{35\,b^{1/4}\,\sqrt{a+b\,x^4}}$$
$$2\,a^{5/4}\,\left(21\,\sqrt{b}\,d+5\,\sqrt{a}\,f\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\Big[2\,\text{ArcTan}\Big[\frac{b^{1/4}\,x}{a^{1/4}}\Big],\,\frac{1}{2}\Big]$$

Result (type 4, 326 leaves) :

$$\frac{1}{420\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^2\,\sqrt{a+b\,x^4}}\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\right.$$
$$\left(\left(a+b\,x^4\right)\,\left(-210\,a\,c+b\,x^4\,\left(105\,c+84\,d\,x+70\,e\,x^2+60\,f\,x^3\right)+20\,a\,x\,\left(-21\,d+x\,\left(14\,e+9\,f\,x\right)\right)\right)+\right.$$
$$\left.315\,a\,\sqrt{b}\,c\,x^2\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\Big[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\Big]-210\,a^{3/2}\,e\,x^2\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\Big[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\Big]\right)+$$
$$1008\,a^{3/2}\,\sqrt{b}\,d\,x^2\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\Big[i\,\text{ArcSinh}\Big[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\Big],\,-1\Big]-$$
$$\left.48\,i\,a^{3/2}\,\left(-21\,i\,\sqrt{b}\,d+5\,\sqrt{a}\,f\right)\,x^2\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\Big[i\,\text{ArcSinh}\Big[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\Big],\,-1\Big]\right)$$

Problem 518: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\,\left(a+b\,x^4\right)^{3/2}}{x^4}\,dx$$

Optimal (type 4, 408 leaves, 16 steps) :

$$\frac{12\, a\, \sqrt{b}\, e\, x\, \sqrt{a+b\, x^4}}{5\, \left(\sqrt{a}+\sqrt{b}\, x^2\right)}-\frac{2\, \left(9\, a\, e-5\, b\, c\, x^2\right)\, \sqrt{a+b\, x^4}}{15\, x}+$$

$$\frac{1}{4}\, \left(2\, a\, f+3\, b\, d\, x^2\right)\, \sqrt{a+b\, x^4}-\frac{\left(5\, c-3\, e\, x^2\right)\, \left(a+b\, x^4\right)^{3/2}}{15\, x^3}-\frac{\left(3\, d-f\, x^2\right)\, \left(a+b\, x^4\right)^{3/2}}{6\, x^2}+$$

$$\frac{3}{4}\, a\, \sqrt{b}\, d\, \text{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b\, x^4}}\right]-\frac{1}{2}\, a^{3/2}\, f\, \text{ArcTanh}\left[\frac{\sqrt{a+b\, x^4}}{\sqrt{a}}\right]-\frac{1}{5\, \sqrt{a+b\, x^4}}$$

$$12\, a^{5/4}\, b^{1/4}\, e\, \left(\sqrt{a}+\sqrt{b}\, x^2\right)\, \sqrt{\frac{a+b\, x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\, \text{EllipticE}\left[2\, \text{ArcTan}\left[\frac{b^{1/4}\, x}{a^{1/4}}\right],\, \frac{1}{2}\right]+\frac{1}{15\, \sqrt{a+b\, x^4}}$$

$$2\, a^{3/4}\, b^{1/4}\, \left(5\, \sqrt{b}\, c+9\, \sqrt{a}\, e\right)\, \left(\sqrt{a}+\sqrt{b}\, x^2\right)\, \sqrt{\frac{a+b\, x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\, \text{EllipticF}\left[2\, \text{ArcTan}\left[\frac{b^{1/4}\, x}{a^{1/4}}\right],\, \frac{1}{2}\right]$$

Result (type 4, 327 leaves):

$$\frac{1}{60\, \sqrt{\frac{i\, \sqrt{b}}{\sqrt{a}}}\, x^3\, \sqrt{a+b\, x^4}}$$

$$\left(\sqrt{\frac{i\, \sqrt{b}}{\sqrt{a}}}\, \left(\left(a+b\, x^4\right)\, \left(-10\, a\, \left(2\, c+x\, \left(3\, d+6\, e\, x-4\, f\, x^2\right)\right)+b\, x^4\, \left(20\, c+x\, \left(15\, d+2\, x\, \left(6\, e+5\, f\, x\right)\right)\right)\right)+\right.\right.$$

$$\left.45\, a\, \sqrt{b}\, d\, x^3\, \sqrt{a+b\, x^4}\, \text{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b\, x^4}}\right]-30\, a^{3/2}\, f\, x^3\, \sqrt{a+b\, x^4}\, \text{ArcTanh}\left[\frac{\sqrt{a+b\, x^4}}{\sqrt{a}}\right]\right)+$$

$$144\, a^{3/2}\, \sqrt{b}\, e\, x^3\, \sqrt{1+\frac{b\, x^4}{a}}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i\, \sqrt{b}}{\sqrt{a}}}\, x\right],\, -1\right]-$$

$$\left.16\, a\, \sqrt{b}\, \left(5\, i\, \sqrt{b}\, c+9\, \sqrt{a}\, e\right)\, x^3\, \sqrt{1+\frac{b\, x^4}{a}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i\, \sqrt{b}}{\sqrt{a}}}\, x\right],\, -1\right]\right)$$

Problem 519: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(c+d\, x+e\, x^2+f\, x^3\right)\, \left(a+b\, x^4\right)^{3/2}}{x^5}\, dx$$

Optimal (type 4, 386 leaves, 15 steps):

$$\frac{12\,a\,\sqrt{b}\,f\,x\,\sqrt{a+b\,x^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{3}{4}\,b\left(c+e\,x^2\right)\sqrt{a+b\,x^4}+$$
$$\frac{2}{15}\,b\,x\left(5\,d+9\,f\,x^2\right)\sqrt{a+b\,x^4}-\frac{1}{12}\left(\frac{3\,c}{x^4}+\frac{4\,d}{x^3}+\frac{6\,e}{x^2}+\frac{12\,f}{x}\right)\left(a+b\,x^4\right)^{3/2}+$$
$$\frac{3}{4}\,a\,\sqrt{b}\,e\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]-\frac{3}{4}\,\sqrt{a}\,b\,c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]-\frac{1}{5\,\sqrt{a+b\,x^4}}$$
$$12\,a^{5/4}\,b^{1/4}\,f\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]+\frac{1}{15\,\sqrt{a+b\,x^4}}$$
$$2\,a^{3/4}\,b^{1/4}\left(5\,\sqrt{b}\,d+9\,\sqrt{a}\,f\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 329 leaves) :

$$\frac{1}{60\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^4\,\sqrt{a+b\,x^4}}$$
$$\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\left(-\left(a+b\,x^4\right)\left(5\,a\left(3\,c+4\,d\,x+6\,x^2\left(e+2\,f\,x\right)\right)-b\,x^4\left(30\,c+x\left(20\,d+3\,x\left(5\,e+4\,f\,x\right)\right)\right)\right)+\right.\right.$$
$$\left.45\,a\,\sqrt{b}\,e\,x^4\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]-45\,\sqrt{a}\,b\,c\,x^4\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+$$
$$144\,a^{3/2}\,\sqrt{b}\,f\,x^4\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-$$
$$\left.16\,a\,\sqrt{b}\left(5\,i\,\sqrt{b}\,d+9\,\sqrt{a}\,f\right)x^4\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 520: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\left(a+b\,x^4\right)^{3/2}}{x^6}\,dx$$

Optimal (type 4, 387 leaves, 15 steps) :

$$\frac{12\,b^{3/2}\,c\,x\,\sqrt{a+b\,x^4}}{5\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{2\,b\,\left(9\,c-5\,e\,x^2\right)\,\sqrt{a+b\,x^4}}{15\,x}+$$

$$\frac{3}{4}\,b\,\left(d+f\,x^2\right)\,\sqrt{a+b\,x^4}-\frac{1}{60}\,\left(\frac{12\,c}{x^5}+\frac{15\,d}{x^4}+\frac{20\,e}{x^3}+\frac{30\,f}{x^2}\right)\,\left(a+b\,x^4\right)^{3/2}+$$

$$\frac{3}{4}\,a\,\sqrt{b}\,f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]-\frac{3}{4}\,\sqrt{a}\,b\,d\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]-\frac{1}{5\,\sqrt{a+b\,x^4}}$$

$$12\,a^{1/4}\,b^{5/4}\,c\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]+\frac{1}{15\,\sqrt{a+b\,x^4}}$$

$$2\,a^{1/4}\,b^{3/4}\,\left(9\,\sqrt{b}\,c+5\,\sqrt{a}\,e\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 331 leaves):

$$\frac{1}{60\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^5\,\sqrt{a+b\,x^4}}$$

$$\left(-\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(\left(a+b\,x^4\right)\,\left(12\,a\,c+84\,b\,c\,x^4+5\,a\,x\,\left(3\,d+4\,e\,x+6\,f\,x^2\right)-5\,b\,x^5\,\left(6\,d+x\,\left(4\,e+3\,f\,x\right)\right)\right)-\right.\right.$$

$$\left.45\,a\,\sqrt{b}\,f\,x^5\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]+45\,\sqrt{a}\,b\,d\,x^5\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+$$

$$144\,\sqrt{a}\,b^{3/2}\,c\,x^5\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-$$

$$\left.16\,i\,\sqrt{a}\,b\,\left(-9\,i\,\sqrt{b}\,c+5\,\sqrt{a}\,e\right)\,x^5\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 521: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\,\left(a+b\,x^4\right)^{3/2}}{x^7}\,dx$$

Optimal (type 4, 392 leaves, 15 steps):

$$\frac{12\, b^{3/2}\, d\, x \sqrt{a+b x^4}}{5 \left(\sqrt{a}+\sqrt{b}\, x^2\right)} - \frac{b \left(2 c - 3 e x^2\right) \sqrt{a+b x^4}}{4 x^2} - \frac{2 b \left(9 d - 5 f x^2\right) \sqrt{a+b x^4}}{15 x} - \frac{1}{60} \left(\frac{10 c}{x^6} + \frac{12 d}{x^5} + \frac{15 e}{x^4} + \frac{20 f}{x^3}\right) \left(a+b x^4\right)^{3/2} +$$

$$\frac{1}{2} b^{3/2} c \,\text{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right] - \frac{3}{4} \sqrt{a}\, b\, e \,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right] - \frac{1}{5 \sqrt{a+b x^4}}$$

$$12\, a^{1/4} b^{5/4} d \left(\sqrt{a}+\sqrt{b}\, x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}} \,\text{EllipticE}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] + \frac{1}{15 \sqrt{a+b x^4}}$$

$$2\, a^{1/4} b^{3/4} \left(9 \sqrt{b}\, d + 5 \sqrt{a}\, f\right) \left(\sqrt{a}+\sqrt{b}\, x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}} \,\text{EllipticF}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 331 leaves):

$$\frac{1}{60 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x^6 \sqrt{a+b x^4}} \left(-\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\right.$$

$$\left(\left(a+b x^4\right) \left(2 b x^4 \left(20 c + x \left(42 d - 5 x \left(3 e + 2 f x\right)\right)\right) + a \left(10 c + x \left(12 d + 5 x \left(3 e + 4 f x\right)\right)\right)\right) -\right.$$

$$\left. 30\, b^{3/2} c\, x^6 \sqrt{a+b x^4} \,\text{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right] + 45 \sqrt{a}\, b\, e\, x^6 \sqrt{a+b x^4} \,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right) +$$

$$144 \sqrt{a}\, b^{3/2} d\, x^6 \sqrt{1+\frac{b x^4}{a}} \,\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x\right], -1\right] -$$

$$\left. 16\, i \sqrt{a}\, b \left(-9\, i \sqrt{b}\, d + 5 \sqrt{a}\, f\right) x^6 \sqrt{1+\frac{b x^4}{a}} \,\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x\right], -1\right]\right)$$

Problem 522: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(c + d x + e x^2 + f x^3\right) \left(a + b x^4\right)^{3/2}}{x^8} \, dx$$

Optimal (type 4, 412 leaves, 16 steps):

$$-\frac{12\,b\,e\,\sqrt{a+b\,x^4}}{5\,x}+\frac{12\,b^{3/2}\,e\,x\,\sqrt{a+b\,x^4}}{5\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{2\,b\left(5\,c-21\,e\,x^2\right)\sqrt{a+b\,x^4}}{35\,x^3}-\frac{b\left(2\,d-3\,f\,x^2\right)\sqrt{a+b\,x^4}}{4\,x^2}-\frac{1}{420}\left(\frac{60\,c}{x^7}+\frac{70\,d}{x^6}+\frac{84\,e}{x^5}+\frac{105\,f}{x^4}\right)\left(a+b\,x^4\right)^{3/2}+\frac{1}{2}\,b^{3/2}\,d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]-\frac{3}{4}\,\sqrt{a}\,b\,f\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]-\frac{1}{5\,\sqrt{a+b\,x^4}}12\,a^{1/4}\,b^{5/4}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]+\frac{1}{35\,a^{1/4}\,\sqrt{a+b\,x^4}}2\,b^{5/4}\left(5\,\sqrt{b}\,c+21\,\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 330 leaves):

$$\frac{1}{420\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^7\,\sqrt{a+b\,x^4}}\left(-\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\left(\left(a+b\,x^4\right)\left(2\,b\,x^4\left(90\,c+7\,x\left(20\,d+3\,x\left(14\,e-5\,f\,x\right)\right)\right)+a\left(60\,c+7\,x\left(10\,d+3\,x\left(4\,e+5\,f\,x\right)\right)\right)\right)-210\,b^{3/2}\,d\,x^7\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]+315\,\sqrt{a}\,b\,f\,x^7\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+1008\,\sqrt{a}\,b^{3/2}\,e\,x^7\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-48\,b^{3/2}\left(5\,i\,\sqrt{b}\,c+21\,\sqrt{a}\,e\right)x^7\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 523: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\left(a+b\,x^4\right)^{3/2}}{x^9}\,dx$$

Optimal (type 4, 377 leaves, 14 steps):

$$-\frac{1}{560} b \left(\frac{105 c}{x^4}+\frac{160 d}{x^3}+\frac{280 e}{x^2}+\frac{672 f}{x}\right) \sqrt{a+b x^4}+$$
$$\frac{12 b^{3/2} f x \sqrt{a+b x^4}}{5 \left(\sqrt{a}+\sqrt{b} x^2\right)}-\frac{1}{840} \left(\frac{105 c}{x^8}+\frac{120 d}{x^7}+\frac{140 e}{x^6}+\frac{168 f}{x^5}\right) \left(a+b x^4\right)^{3/2}+$$
$$\frac{1}{2} b^{3/2} e \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]-\frac{3 b^2 c \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{16 \sqrt{a}}-\frac{1}{5 \sqrt{a+b x^4}}$$
$$12 a^{1/4} b^{5/4} f \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]+\frac{1}{35 a^{1/4} \sqrt{a+b x^4}}$$
$$2 b^{5/4} \left(5 \sqrt{b} d+21 \sqrt{a} f\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 309 leaves):

$$-\frac{1}{1680 x^8}$$
$$\sqrt{a+b x^4} \left(b x^4 \left(525 c+16 x \left(45 d+70 e x+147 f x^2\right)\right)+a \left(210 c+8 x \left(30 d+7 x \left(5 e+6 f x\right)\right)\right)\right)+$$
$$\frac{1}{2} b^{3/2} e \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]-\frac{3 b^2 c \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{16 \sqrt{a}}-$$
$$\frac{12 i a \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b f \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]}{5 \sqrt{a+b x^4}}-$$
$$\left(4 b^{3/2} \left(5 i \sqrt{b} d+21 \sqrt{a} f\right) \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right) \Big/$$
$$\left(35 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \sqrt{a+b x^4}\right)$$

Problem 524: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(c+d x+e x^2+f x^3\right) \left(a+b x^4\right)^{3/2}}{x^{10}} \, dx$$

Optimal (type 4, 405 leaves, 15 steps):

$$-\frac{b\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\sqrt{a+bx^4}}{1680}-\frac{4b^2c\sqrt{a+bx^4}}{15ax}+\frac{4b^{5/2}cx\sqrt{a+bx^4}}{15a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{1}{504}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)\left(a+bx^4\right)^{3/2}+\frac{1}{2}b^{3/2}f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right]-\frac{3b^2d\,\text{ArcTanh}\left[\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right]}{16\sqrt{a}}-\frac{1}{15a^{3/4}\sqrt{a+bx^4}}4b^{9/4}c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]+\left(2b^{7/4}\left(7\sqrt{b}\,c+15\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(105a^{3/4}\sqrt{a+bx^4}\right)$$

Result (type 4, 351 leaves):

$$\frac{1}{5040a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^9\sqrt{a+bx^4}}\left(-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+bx^4\right)\left(1344b^2cx^8+10a^2\left(56c+63dx+72ex^2+84fx^3\right)+abx^4\left(1232c+15x\left(105d+16x\left(9e+14fx\right)\right)\right)\right)-2520ab^{3/2}fx^9\sqrt{a+bx^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right]+945\sqrt{a}\,b^2dx^9\sqrt{a+bx^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right]\right)+1344\sqrt{a}\,b^{5/2}cx^9\sqrt{1+\frac{bx^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-192i\sqrt{a}\,b^2\left(-7i\sqrt{b}\,c+15\sqrt{a}\,e\right)x^9\sqrt{1+\frac{bx^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 525: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}}{x^{11}}\,dx$$

Optimal (type 4, 399 leaves, 13 steps):

$$-\frac{b\left(\frac{168 c}{x^6}+\frac{224 d}{x^5}+\frac{315 e}{x^4}+\frac{480 f}{x^3}\right)\sqrt{a+b x^4}}{1680}-\frac{b^2 c\sqrt{a+b x^4}}{10 a x^2}-\frac{4 b^2 d\sqrt{a+b x^4}}{15 a x}+\frac{4 b^{5/2} d x\sqrt{a+b x^4}}{15 a\left(\sqrt{a}+\sqrt{b} x^2\right)}-\frac{\left(\frac{252 c}{x^{10}}+\frac{280 d}{x^9}+\frac{315 e}{x^8}+\frac{360 f}{x^7}\right)\left(a+b x^4\right)^{3/2}}{2520}-\frac{3 b^2 e \text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{16\sqrt{a}}-\frac{1}{15 a^{3/4}\sqrt{a+b x^4}}4 b^{9/4} d\left(\sqrt{a}+\sqrt{b} x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]+\left(2 b^{7/4}\left(7\sqrt{b} d+15\sqrt{a} f\right)\left(\sqrt{a}+\sqrt{b} x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(105 a^{3/4}\sqrt{a+b x^4}\right)$$

Result (type 4, 314 leaves):

$$\frac{1}{5040 a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} x^{10}\sqrt{a+b x^4}}\left(-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+b x^4\right)\left(168 b^2 x^8(3 c+8 d x)+a^2(504 c+10 x(56 d+9 x(7 e+8 f x)))+a b x^4(1008 c+x(1232 d+45 x(35 e+48 f x)))\right)+945\sqrt{a} b^2 e x^{10}\sqrt{a+b x^4} \text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right)+1344\sqrt{a} b^{5/2} d x^{10}\sqrt{1+\frac{b x^4}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} x\right],-1\right]-192 i\sqrt{a} b^2\left(-7 i\sqrt{b} d+15\sqrt{a} f\right) x^{10}\sqrt{1+\frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right)$$

Problem 526: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d x+e x^2+f x^3\right)\left(a+b x^4\right)^{3/2}}{x^{12}}\,dx$$

Optimal (type 4, 424 leaves, 14 steps):

$$-\frac{b\left(\frac{1440\,c}{x^7}+\frac{1848\,d}{x^6}+\frac{2464\,e}{x^5}+\frac{3465\,f}{x^4}\right)\sqrt{a+b\,x^4}}{18\,480}-\frac{4\,b^2\,c\,\sqrt{a+b\,x^4}}{77\,a\,x^3}-\frac{b^2\,d\,\sqrt{a+b\,x^4}}{10\,a\,x^2}-\frac{4\,b^2\,e\,\sqrt{a+b\,x^4}}{15\,a\,x}+$$

$$\frac{4\,b^{5/2}\,e\,x\,\sqrt{a+b\,x^4}}{15\,a\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{\left(\frac{360\,c}{x^{11}}+\frac{396\,d}{x^{10}}+\frac{440\,e}{x^9}+\frac{495\,f}{x^8}\right)\left(a+b\,x^4\right)^{3/2}}{3960}-\frac{3\,b^2\,f\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{16\,\sqrt{a}}-$$

$$\frac{1}{15\,a^{3/4}\,\sqrt{a+b\,x^4}}4\,b^{9/4}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]-$$

$$\left(2\,b^{9/4}\left(15\,\sqrt{b}\,c-77\,\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(1155\,a^{5/4}\,\sqrt{a+b\,x^4}\right)$$

Result (type 4, 317 leaves):

$$\frac{1}{55\,440\,a\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x^{11}\,\sqrt{a+b\,x^4}}\left(-\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\right.$$

$$\left(\left(a+b\,x^4\right)\left(24\,b^2\,x^8\left(120\,c+77\,x\left(3\,d+8\,e\,x\right)\right)+a\,b\,x^4\left(9360\,c+77\,x\left(144\,d+176\,e\,x+225\,f\,x^2\right)\right)+\right.\right.$$

$$\left.14\,a^2\left(360\,c+11\,x\left(36\,d+5\,x\left(8\,e+9\,f\,x\right)\right)\right)\right)+$$

$$\left.10\,395\,\sqrt{a}\,b^2\,f\,x^{11}\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+$$

$$14\,784\,\sqrt{a}\,b^{5/2}\,e\,x^{11}\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-$$

$$\left.192\,b^{5/2}\left(-15\,i\,\sqrt{b}\,c+77\,\sqrt{a}\,e\right)x^{11}\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 527: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d\,x+e\,x^2+f\,x^3\right)\left(a+b\,x^4\right)^{3/2}}{x^{13}}\,dx$$

Optimal (type 4, 449 leaves, 15 steps):

$$-\frac{b\left(\frac{1155 c}{x^8}+\frac{1440 d}{x^7}+\frac{1848 e}{x^6}+\frac{2464 f}{x^5}\right)\sqrt{a+b x^4}}{18480}-\frac{b^2 c\sqrt{a+b x^4}}{32 a x^4}-\frac{4 b^2 d\sqrt{a+b x^4}}{77 a x^3}-\frac{b^2 e\sqrt{a+b x^4}}{10 a x^2}-\frac{4 b^2 f\sqrt{a+b x^4}}{15 a x}+\frac{4 b^{5/2} f x\sqrt{a+b x^4}}{15 a\left(\sqrt{a}+\sqrt{b}\, x^2\right)}-\frac{\left(\frac{165 c}{x^{12}}+\frac{180 d}{x^{11}}+\frac{198 e}{x^{10}}+\frac{220 f}{x^9}\right)\left(a+b x^4\right)^{3/2}}{1980}+\frac{b^3 c\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{32 a^{3/2}}-\frac{1}{15 a^{3/4}\sqrt{a+b x^4}}4 b^{9/4} f\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]-\left(2 b^{9/4}\left(15\sqrt{b}\, d-77\sqrt{a}\, f\right)\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(1155 a^{5/4}\sqrt{a+b x^4}\right)$$

Result (type 4, 328 leaves):

$$\frac{1}{110880 a^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x^{12}\sqrt{a+b x^4}}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\sqrt{a}\left(a+b x^4\right)\left(56 a^2\left(165 c+2 x\left(90 d+99 e x+110 f x^2\right)\right)+3 b^2 x^8\left(1155 c+16 x\left(120 d+77 x\left(3 e+8 f x\right)\right)\right)+2 a b x^4\left(8085 c+16 x\left(585 d+77 x\left(9 e+11 f x\right)\right)\right)\right)+3465 b^3 c x^{12}\sqrt{a+b x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right)+29568 a b^{5/2} f x^{12}\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]-384\sqrt{a}\, b^{5/2}\left(-15 i\sqrt{b}\, d+77\sqrt{a}\, f\right)x^{12}\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]\right)$$

Problem 528: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(c+d x+e x^2+f x^3\right)\left(a+b x^4\right)^{3/2}}{x^{14}}\,dx$$

Optimal (type 4, 474 leaves, 16 steps):

$$-\frac{b\left(\frac{12320\,c}{x^9}+\frac{15015\,d}{x^8}+\frac{18720\,e}{x^7}+\frac{24024\,f}{x^6}\right)\sqrt{a+b\,x^4}}{240240}-\frac{4\,b^2\,c\,\sqrt{a+b\,x^4}}{195\,a\,x^5}-\frac{b^2\,d\,\sqrt{a+b\,x^4}}{32\,a\,x^4}-\frac{4\,b^2\,e\,\sqrt{a+b\,x^4}}{77\,a\,x^3}-\frac{b^2\,f\,\sqrt{a+b\,x^4}}{10\,a\,x^2}+\frac{4\,b^3\,c\,\sqrt{a+b\,x^4}}{65\,a^2\,x}-\frac{4\,b^{7/2}\,c\,x\,\sqrt{a+b\,x^4}}{65\,a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{\left(\frac{660\,c}{x^{13}}+\frac{715\,d}{x^{12}}+\frac{780\,e}{x^{11}}+\frac{858\,f}{x^{10}}\right)\left(a+b\,x^4\right)^{3/2}}{8580}+\frac{b^3\,d\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{32\,a^{3/2}}+\frac{1}{65\,a^{7/4}\sqrt{a+b\,x^4}}4\,b^{13/4}\,c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\mathrm{EllipticE}\left[2\,\mathrm{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]-\left(2\,b^{11/4}\left(77\sqrt{b}\,c+65\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(5005\,a^{7/4}\sqrt{a+b\,x^4}\right)$$

Result (type 4, 339 leaves):

$$\frac{1}{480480\,a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^{13}\sqrt{a+b\,x^4}}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\left(a+b\,x^4\right)\left(-29568\,b^3\,c\,x^{12}+56\,a^3\left(660\,c+13\,x\left(55\,d+60\,e\,x+66\,f\,x^2\right)\right)+a\,b^2\,x^8\left(9856\,c+39\,x\left(385\,d+16\,x\left(40\,e+77\,f\,x\right)\right)\right)+2\,a^2\,b\,x^4\left(30800\,c+13\,x\left(2695\,d+48\,x\left(65\,e+77\,f\,x\right)\right)\right)\right)+15015\sqrt{a}\,b^3\,d\,x^{13}\sqrt{a+b\,x^4}\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)-29568\sqrt{a}\,b^{7/2}\,c\,x^{13}\sqrt{1+\frac{b\,x^4}{a}}\,\mathrm{EllipticE}\left[i\,\mathrm{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+384\sqrt{a}\,b^3\left(77\sqrt{b}\,c+65\,i\sqrt{a}\,e\right)x^{13}\sqrt{1+\frac{b\,x^4}{a}}\,\mathrm{EllipticF}\left[i\,\mathrm{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 529: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 \left(c + d x + e x^2 + f x^3\right)}{\sqrt{a + b x^4}} \, dx$$

Optimal (type 4, 361 leaves, 12 steps):

$$\frac{c x \sqrt{a+b x^4}}{3 b} + \frac{e x^3 \sqrt{a+b x^4}}{5 b} + \frac{f x^4 \sqrt{a+b x^4}}{6 b} - \frac{3 a e x \sqrt{a+b x^4}}{5 b^{3/2} \left(\sqrt{a}+\sqrt{b} x^2\right)} -$$
$$\frac{\left(4 a f - 3 b d x^2\right) \sqrt{a+b x^4}}{12 b^2} - \frac{a d \,\text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{4 b^{3/2}} + \frac{1}{5 b^{7/4} \sqrt{a+b x^4}}$$
$$3 a^{5/4} e \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \,\text{EllipticE}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] - \frac{1}{30 b^{7/4} \sqrt{a+b x^4}}$$
$$a^{3/4} \left(5 \sqrt{b} c + 9 \sqrt{a} e\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \,\text{EllipticF}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 259 leaves):

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \left(-\left(a+b x^4\right) \left(20 a f - b x \left(20 c + x \left(15 d + 2 x \left(6 e + 5 f x\right)\right)\right)\right) -\right.\right.$$
$$\left. 15 a \sqrt{b} d \sqrt{a+b x^4} \,\text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]\right) -$$
$$36 a^{3/2} \sqrt{b} e \sqrt{1+\frac{b x^4}{a}} \,\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right] +$$
$$\left. 4 a \sqrt{b} \left(5 i \sqrt{b} c + 9 \sqrt{a} e\right) \sqrt{1+\frac{b x^4}{a}} \,\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]\right) \Big/$$
$$\left(60 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b^2 \sqrt{a+b x^4}\right)$$

Problem 530: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 \left(c + d x + e x^2 + f x^3\right)}{\sqrt{a + b x^4}} \, dx$$

Optimal (type 4, 336 leaves, 11 steps):

$$\frac{d\,x\,\sqrt{a+b\,x^4}}{3\,b}+\frac{f\,x^3\,\sqrt{a+b\,x^4}}{5\,b}-\frac{3\,a\,f\,x\,\sqrt{a+b\,x^4}}{5\,b^{3/2}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+$$

$$\frac{\left(2\,c+e\,x^2\right)\,\sqrt{a+b\,x^4}}{4\,b}-\frac{a\,e\,\mathrm{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{4\,b^{3/2}}+\frac{1}{5\,b^{7/4}\,\sqrt{a+b\,x^4}}$$

$$3\,a^{5/4}\,f\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\mathrm{EllipticE}\left[2\,\mathrm{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]-\frac{1}{30\,b^{7/4}\,\sqrt{a+b\,x^4}}$$

$$a^{3/4}\,\left(5\,\sqrt{b}\,d+9\,\sqrt{a}\,f\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]$$

Result (type 4, 241 leaves) :

$$\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\right.$$

$$\left(\sqrt{b}\,\left(a+b\,x^4\right)\,\left(30\,c+x\,\left(20\,d+3\,x\,\left(5\,e+4\,f\,x\right)\right)\right)-15\,a\,e\,\sqrt{a+b\,x^4}\,\mathrm{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]\right)-$$

$$36\,a^{3/2}\,f\,\sqrt{1+\frac{b\,x^4}{a}}\,\mathrm{EllipticE}\left[i\,\mathrm{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]+$$

$$\left.4\,a\,\left(5\,i\,\sqrt{b}\,d+9\,\sqrt{a}\,f\right)\,\sqrt{1+\frac{b\,x^4}{a}}\,\mathrm{EllipticF}\left[i\,\mathrm{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]\right)\Big/$$

$$\left(60\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,b^{3/2}\,\sqrt{a+b\,x^4}\right)$$

Problem 531: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\,\left(c+d\,x+e\,x^2+f\,x^3\right)}{\sqrt{a+b\,x^4}}\,dx$$

Optimal (type 4, 308 leaves, 10 steps) :

$$\frac{e\,x\,\sqrt{a+b\,x^4}}{3\,b}+\frac{c\,x\,\sqrt{a+b\,x^4}}{\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{\left(2\,d+f\,x^2\right)\,\sqrt{a+b\,x^4}}{4\,b}-\frac{a\,f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{4\,b^{3/2}}-$$

$$\frac{a^{1/4}\,c\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{b^{3/4}\,\sqrt{a+b\,x^4}}+\frac{1}{6\,b^{5/4}\,\sqrt{a+b\,x^4}}$$

$$a^{1/4}\,\left(3\,\sqrt{b}\,c-\sqrt{a}\,e\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]$$

Result (type 4, 245 leaves):

$$\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(\sqrt{b}\,\left(6\,d+4\,e\,x+3\,f\,x^2\right)\,\left(a+b\,x^4\right)-3\,a\,f\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]\right)+\right.$$

$$12\,\sqrt{a}\,b\,c\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]+4\,i\,\sqrt{a}\,\sqrt{b}\,\left(3\,i\,\sqrt{b}\,c+\sqrt{a}\,e\right)$$

$$\left.\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]\right)\Bigg/\left(12\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,b^{3/2}\,\sqrt{a+b\,x^4}\right)$$

Problem 532: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\,\left(c+d\,x+e\,x^2+f\,x^3\right)}{\sqrt{a+b\,x^4}}\,dx$$

Optimal (type 4, 299 leaves, 10 steps):

$$\frac{e\,\sqrt{a+b\,x^4}}{2\,b}+\frac{f\,x\,\sqrt{a+b\,x^4}}{3\,b}+\frac{d\,x\,\sqrt{a+b\,x^4}}{\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{c\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b\,x^4}}\right]}{2\,\sqrt{b}}-$$

$$\frac{a^{1/4}\,d\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{b^{3/4}\,\sqrt{a+b\,x^4}}+\frac{1}{6\,b^{5/4}\,\sqrt{a+b\,x^4}}$$

$$a^{1/4}\,\left(3\,\sqrt{b}\,d-\sqrt{a}\,f\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]$$

Result (type 4, 235 leaves):

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left((3e+2fx)\left(a+bx^4\right)+3\sqrt{b}\,c\sqrt{a+bx^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right]\right)+\right.$$
$$6\sqrt{a}\sqrt{b}\,d\sqrt{1+\frac{bx^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+2\,i\sqrt{a}\left(3\,i\sqrt{b}\,d+\sqrt{a}\,f\right)$$
$$\left.\sqrt{1+\frac{bx^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)\Bigg/\left(6\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,b\sqrt{a+bx^4}\right)$$

Problem 533: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx$$

Optimal (type 4, 276 leaves, 9 steps):

$$\frac{f\sqrt{a+bx^4}}{2b}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right]}{2\sqrt{b}}-$$
$$\frac{a^{1/4}e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{b^{3/4}\sqrt{a+bx^4}}+\frac{1}{2\,b^{3/4}\sqrt{a+bx^4}}$$
$$a^{1/4}\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 225 leaves):

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(af+bfx^4+\sqrt{b}\,d\sqrt{a+bx^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+bx^4}}\right]\right)+\right.$$
$$2\sqrt{a}\sqrt{b}\,e\sqrt{1+\frac{bx^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-2\sqrt{b}\left(i\sqrt{b}\,c+\sqrt{a}\,e\right)$$
$$\left.\sqrt{1+\frac{bx^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)\Bigg/\left(2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,b\sqrt{a+bx^4}\right)$$

Problem 534: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx$$

Optimal (type 4, 285 leaves, 12 steps):

$$\frac{f x \sqrt{a+b x^4}}{\sqrt{b} \left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{e \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{2 \sqrt{b}}-\frac{c \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{2 \sqrt{a}}-\frac{a^{1/4} f \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{b^{3/4} \sqrt{a+b x^4}}+\frac{1}{2 b^{3/4} \sqrt{a+b x^4}} a^{1/4} \left(\frac{\sqrt{b} d}{\sqrt{a}}+f\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 235 leaves):

$$-\frac{1}{2 b \sqrt{a+b x^4}} i \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \sqrt{a+b x^4} \left(\sqrt{a} e \operatorname{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]-\sqrt{b} c \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right)+2 a f \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]-2 \sqrt{a} \left(i \sqrt{b} d+\sqrt{a} f\right) \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]\right)$$

Problem 535: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c+d x+e x^2+f x^3}{x^2 \sqrt{a+b x^4}} \, dx$$

Optimal (type 4, 309 leaves, 13 steps):

$$-\frac{c\sqrt{a+b x^4}}{a x}+\frac{\sqrt{b}\, c x\sqrt{a+b x^4}}{a\left(\sqrt{a}+\sqrt{b}\, x^2\right)}+\frac{f\,\text{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]}{2\sqrt{b}}-\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{2\sqrt{a}}-$$

$$\frac{b^{1/4} c\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{a^{3/4}\sqrt{a+b x^4}}+$$

$$\left(\left(\sqrt{b}\, c+\sqrt{a}\, e\right)\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(2 a^{3/4} b^{1/4}\sqrt{a+b x^4}\right)$$

Result (type 4, 250 leaves) :

$$\frac{1}{2}\left(-\frac{2 c\sqrt{a+b x^4}}{a x}+\frac{f\,\text{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]}{\sqrt{b}}-\frac{d\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{\sqrt{a}}\right)-$$

$$\frac{i\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, c\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]}{\sqrt{a+b x^4}}-\frac{1}{\sqrt{b}\sqrt{a+b x^4}}$$

$$\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-i\sqrt{b}\, c+\sqrt{a}\, e\right)\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]$$

Problem 536: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d x+e x^2+f x^3}{x^3\sqrt{a+b x^4}}\,dx$$

Optimal (type 4, 300 leaves, 11 steps) :

$$-\frac{c\sqrt{a+b x^4}}{2 a x^2}-\frac{d\sqrt{a+b x^4}}{a x}+\frac{\sqrt{b}\, d x\sqrt{a+b x^4}}{a\left(\sqrt{a}+\sqrt{b}\, x^2\right)}-\frac{e \,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{2\sqrt{a}}-$$

$$\frac{b^{1/4} d\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{a^{3/4}\sqrt{a+b x^4}}+$$

$$\left(\left(\sqrt{b}\, d+\sqrt{a}\, f\right)\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(2 a^{3/4} b^{1/4}\sqrt{a+b x^4}\right)$$

Result (type 4, 242 leaves):

$$\left(-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(c+2 d x\right)\left(a+b x^4\right)+\sqrt{a}\, e x^2\sqrt{a+b x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right)+\right.$$

$$2\sqrt{a}\sqrt{b}\, d x^2\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]-2 i\sqrt{a}\left(-i\sqrt{b}\, d+\sqrt{a}\, f\right)$$

$$\left.x^2\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]\right)\Big/\left(2 a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x^2\sqrt{a+b x^4}\right)$$

Problem 537: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d x+e x^2+f x^3}{x^4\sqrt{a+b x^4}}\,dx$$

Optimal (type 4, 323 leaves, 12 steps):

$$-\frac{c\sqrt{a+b x^4}}{3 a x^3}-\frac{d\sqrt{a+b x^4}}{2 a x^2}-\frac{e\sqrt{a+b x^4}}{a x}+\frac{\sqrt{b}\, e x\sqrt{a+b x^4}}{a\left(\sqrt{a}+\sqrt{b}\, x^2\right)}-\frac{f\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{2\sqrt{a}}-$$

$$\frac{b^{1/4} e\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{a^{3/4}\sqrt{a+b x^4}}-\frac{1}{6 a^{5/4}\sqrt{a+b x^4}}$$

$$b^{1/4}\left(\sqrt{b}\, c-3\sqrt{a}\, e\right)\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 249 leaves):

$$\left(-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\left(a+b x^4\right)\left(2 c+3 x\left(d+2 e x\right)\right)+3\sqrt{a}\, f x^3\sqrt{a+b x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right)+6\sqrt{a}\sqrt{b}\, e x^3\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]-2\sqrt{b}\left(-i\sqrt{b}\, c+3\sqrt{a}\, e\right) x^3\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]\right)\Big/\left(6 a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x^3\sqrt{a+b x^4}\right)$$

Problem 538: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d x+e x^2+f x^3}{x^5\sqrt{a+b x^4}}\,dx$$

Optimal (type 4, 346 leaves, 13 steps):

$$-\frac{c\sqrt{a+b x^4}}{4 a x^4}-\frac{d\sqrt{a+b x^4}}{3 a x^3}-\frac{e\sqrt{a+b x^4}}{2 a x^2}-\frac{f\sqrt{a+b x^4}}{a x}+\frac{\sqrt{b}\, f x\sqrt{a+b x^4}}{a\left(\sqrt{a}+\sqrt{b}\, x^2\right)}+\frac{b c\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{4 a^{3/2}}-\frac{b^{1/4} f\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{a^{3/4}\sqrt{a+b x^4}}-\frac{1}{6 a^{5/4}\sqrt{a+b x^4}}b^{1/4}\left(\sqrt{b}\, d-3\sqrt{a}\, f\right)\left(\sqrt{a}+\sqrt{b}\, x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 259 leaves):

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\sqrt{a}\left(a+b x^4\right)\left(3 c+4 d x+6 x^2\left(e+2 f x\right)\right)+3 b c x^4\sqrt{a+b x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right)+12 a\sqrt{b}\, f x^4\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]-4\sqrt{a}\sqrt{b}\left(-i\sqrt{b}\, d+3\sqrt{a}\, f\right) x^4\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]\right)\Big/\left(12 a^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\, x^4\sqrt{a+b x^4}\right)$$

Problem 539: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c + d x + e x^2 + f x^3}{x^6 \sqrt{a + b x^4}} \, dx$$

Optimal (type 4, 377 leaves, 14 steps):

$$-\frac{c \sqrt{a+b x^4}}{5 a x^5} - \frac{d \sqrt{a+b x^4}}{4 a x^4} - \frac{e \sqrt{a+b x^4}}{3 a x^3} - \frac{f \sqrt{a+b x^4}}{2 a x^2} + \frac{3 b c \sqrt{a+b x^4}}{5 a^2 x} - \frac{3 b^{3/2} c x \sqrt{a+b x^4}}{5 a^2 \left(\sqrt{a}+\sqrt{b} x^2\right)} + \frac{b d \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{4 a^{3/2}} + \frac{1}{5 a^{7/4} \sqrt{a+b x^4}} 3 b^{5/4} c \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right] - \frac{1}{30 a^{7/4} \sqrt{a+b x^4}} b^{3/4} \left(9 \sqrt{b} c + 5 \sqrt{a} e\right) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 268 leaves):

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \left(-\left(a+b x^4\right) \left(12 a c - 36 b c x^4 + 5 a x \left(3 d + 4 e x + 6 f x^2\right)\right) + 15 \sqrt{a} b d x^5 \sqrt{a+b x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]\right) - 36 \sqrt{a} b^{3/2} c x^5 \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right] + 4 \sqrt{a} b \left(9 \sqrt{b} c + 5 i \sqrt{a} e\right) x^5 \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right], -1\right]\right) \Bigg/ \left(60 a^2 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x^5 \sqrt{a+b x^4}\right)$$

Problem 540: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6 \left(c + d x + e x^2 + f x^3\right)}{\left(a + b x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 365 leaves, 12 steps):

$$\frac{x\left(a e+a f x-b c x^2-b d x^3\right)}{2 b^2 \sqrt{a+b x^4}}+\frac{d \sqrt{a+b x^4}}{b^2}+\frac{e x \sqrt{a+b x^4}}{3 b^2}+$$

$$\frac{f x^2 \sqrt{a+b x^4}}{4 b^2}+\frac{3 c x \sqrt{a+b x^4}}{2 b^{3/2}\left(\sqrt{a}+\sqrt{b} x^2\right)}-\frac{3 a f \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{4 b^{5/2}}-\frac{1}{2 b^{7/4} \sqrt{a+b x^4}}$$

$$3 a^{1/4} c\left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]+\frac{1}{12 b^{9/4} \sqrt{a+b x^4}}$$

$$a^{1/4}\left(9 \sqrt{b} c-5 \sqrt{a} e\right)\left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 267 leaves):

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\left(\sqrt{b}\left(a\left(12 d+10 e x+9 f x^2\right)+b x^3\left(-6 c+6 d x+4 e x^2+3 f x^3\right)\right)-\right.\right.$$

$$\left.9 a f \sqrt{a+b x^4} \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]\right)+$$

$$18 \sqrt{a} b c \sqrt{1+\frac{b x^4}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]+$$

$$\left.2 i \sqrt{a} \sqrt{b}\left(9 i \sqrt{b} c+5 \sqrt{a} e\right) \sqrt{1+\frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right) \Bigg/$$

$$\left(12 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b^{5/2} \sqrt{a+b x^4}\right)$$

Problem 541: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^5\left(c+d x+e x^2+f x^3\right)}{\left(a+b x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 343 leaves, 11 steps):

$$\frac{x\left(a f-b c x-b d x^2-b e x^3\right)}{2 b^2 \sqrt{a+b x^4}}+\frac{e \sqrt{a+b x^4}}{b^2}+\frac{f x \sqrt{a+b x^4}}{3 b^2}+$$

$$\frac{3 d x \sqrt{a+b x^4}}{2 b^{3/2}\left(\sqrt{a}+\sqrt{b}\, x^2\right)}+\frac{c \operatorname{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]}{2 b^{3/2}}-\frac{1}{2 b^{7/4} \sqrt{a+b x^4}}$$

$$3 a^{1/4} d\left(\sqrt{a}+\sqrt{b}\, x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]+\frac{1}{12 b^{9/4} \sqrt{a+b x^4}}$$

$$a^{1/4}\left(9 \sqrt{b}\, d-5 \sqrt{a}\, f\right)\left(\sqrt{a}+\sqrt{b}\, x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 255 leaves):

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\right.$$

$$\left(a\left(6 e+5 f x\right)+b x^2\left(-3 c-3 d x+3 e x^2+2 f x^3\right)+3 \sqrt{b}\, c \sqrt{a+b x^4} \operatorname{ArcTanh}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a+b x^4}}\right]\right)+$$

$$9 \sqrt{a} \sqrt{b}\, d \sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x\right], -1\right]+i \sqrt{a}\left(9 i \sqrt{b}\, d+5 \sqrt{a}\, f\right)$$

$$\left.\sqrt{1+\frac{b x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x\right], -1\right]\right) \Big/ \left(6 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, b^2 \sqrt{a+b x^4}\right)$$

Problem 542: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4\left(c+d x+e x^2+f x^3\right)}{\left(a+b x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 314 leaves, 10 steps):

$$-\frac{x\left(c+d x+e x^2+f x^3\right)}{2 b \sqrt{a+b x^4}}+\frac{f \sqrt{a+b x^4}}{b^2}+\frac{3 e x \sqrt{a+b x^4}}{2 b^{3/2}\left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{d \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{2 b^{3/2}}-\frac{1}{2 b^{7/4} \sqrt{a+b x^4}} 3 a^{1/4} e\left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]+\left(\left(\sqrt{b} c+3 \sqrt{a} e\right)\left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(4 a^{1/4} b^{7/4} \sqrt{a+b x^4}\right)$$

Result (type 4, 243 leaves):

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\left(2 a f+b x\left(-c-d x-e x^2+f x^3\right)+\sqrt{b} d \sqrt{a+b x^4} \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]\right)+3 \sqrt{a} \sqrt{b} e \sqrt{1+\frac{b x^4}{a}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]-\sqrt{b}\left(i \sqrt{b} c+3 \sqrt{a} e\right) \sqrt{1+\frac{b x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} x\right],-1\right]\right) \Big/ \left(2 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} b^2 \sqrt{a+b x^4}\right)$$

Problem 543: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3\left(c+d x+e x^2+f x^3\right)}{\left(a+b x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 297 leaves, 9 steps):

$$-\frac{c+d x+e x^2+f x^3}{2 b \sqrt{a+b x^4}}+\frac{3 f x \sqrt{a+b x^4}}{2 b^{3/2}\left(\sqrt{a}+\sqrt{b} x^2\right)}+\frac{e \text{ArcTanh}\left[\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right]}{2 b^{3/2}}-\frac{1}{2 b^{7/4} \sqrt{a+b x^4}} 3 a^{1/4} f\left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]+\left(\left(\sqrt{b} d+3 \sqrt{a} f\right)\left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(4 a^{1/4} b^{7/4} \sqrt{a+b x^4}\right)$$

Result (type 4, 224 leaves):

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(-\sqrt{b}\left(c+x\left(d+x\left(e+f x\right)\right)\right)+e\sqrt{a+b x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b x^4}}\right]\right)+3\sqrt{a}\,f\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-\left(i\sqrt{b}\,d+3\sqrt{a}\,f\right)\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)\Bigg/\left(2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,b^{3/2}\sqrt{a+b x^4}\right)$$

Problem 544: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\left(c+d x+e x^2+f x^3\right)}{\left(a+b x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 333 leaves, 10 steps):

$$-\frac{x\left(a e+a f x-b c x^2-b d x^3\right)}{2 a b\sqrt{a+b x^4}}-\frac{d\sqrt{a+b x^4}}{2 a b}-\frac{c x\sqrt{a+b x^4}}{2 a\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+\frac{f\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b x^4}}\right]}{2 b^{3/2}}+\frac{c\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 a^{3/4} b^{3/4}\sqrt{a+b x^4}}-\left(\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Bigg/\left(4 a^{3/4} b^{5/4}\sqrt{a+b x^4}\right)$$

Result (type 4, 242 leaves):

$$\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(\sqrt{b}\left(b c x^3-a\left(d+x\left(e+f x\right)\right)\right)+a f\sqrt{a+b x^4}\,\text{ArcTanh}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a+b x^4}}\right]\right)-\sqrt{a}\,b\,c\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+\sqrt{a}\,\sqrt{b}\left(\sqrt{b}\,c-i\sqrt{a}\,e\right)\sqrt{1+\frac{b x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)\Bigg/\left(2 a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,b^{3/2}\sqrt{a+b x^4}\right)$$

Problem 545: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x\,\left(c+d\,x+e\,x^2+f\,x^3\right)}{\left(a+b\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 303 leaves, 7 steps):

$$-\frac{x\,\left(a\,f-b\,c\,x-b\,d\,x^2-b\,e\,x^3\right)}{2\,a\,b\,\sqrt{a+b\,x^4}}-\frac{e\,\sqrt{a+b\,x^4}}{2\,a\,b}-\frac{d\,x\,\sqrt{a+b\,x^4}}{2\,a\,\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}+$$

$$\frac{d\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{3/4}\,b^{3/4}\,\sqrt{a+b\,x^4}}-$$

$$\left(\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(4\,a^{3/4}\,b^{5/4}\,\sqrt{a+b\,x^4}\right)$$

Result (type 4, 197 leaves):

$$\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(b\,x^2\,\left(c+d\,x\right)-a\,\left(e+f\,x\right)\right)-\right.$$

$$\sqrt{a}\,\sqrt{b}\,d\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]+\sqrt{a}\,\left(\sqrt{b}\,d-i\,\sqrt{a}\,f\right)$$

$$\left.\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)\Big/\left(2\,a\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,b\,\sqrt{a+b\,x^4}\right)$$

Problem 546: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c+d\,x+e\,x^2+f\,x^3}{\left(a+b\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 275 leaves, 4 steps):

$$-\frac{e\,x\,\sqrt{a+b\,x^4}}{2\,a\,\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{a\,f-b\,x\,\left(c+d\,x+e\,x^2\right)}{2\,a\,b\,\sqrt{a+b\,x^4}}+$$

$$\frac{e\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{2\,a^{3/4}\,b^{3/4}\,\sqrt{a+b\,x^4}}+$$

$$\left(\left(\sqrt{b}\,c-\sqrt{a}\,e\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]\right)\Bigg/$$

$$\left(4\,a^{5/4}\,b^{3/4}\,\sqrt{a+b\,x^4}\right)$$

Result (type 4, 195 leaves):

$$\left(\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,\left(-a\,f+b\,x\,\left(c+x\,\left(d+e\,x\right)\right)\right)-\right.$$

$$\sqrt{a}\,\sqrt{b}\,e\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]+\sqrt{b}\,\left(-i\,\sqrt{b}\,c+\sqrt{a}\,e\right)$$

$$\left.\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,x\right],\,-1\right]\right)\Bigg/\left(2\,a\,\sqrt{\frac{i\,\sqrt{b}}{\sqrt{a}}}\,b\,\sqrt{a+b\,x^4}\right)$$

Problem 547: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x+e\,x^2+f\,x^3}{x\,\left(a+b\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 323 leaves, 11 steps):

$$\frac{x\,\left(a\,d+a\,e\,x+a\,f\,x^2-b\,c\,x^3\right)}{2\,a^2\,\sqrt{a+b\,x^4}}+\frac{c\,\sqrt{a+b\,x^4}}{2\,a^2}-\frac{f\,x\,\sqrt{a+b\,x^4}}{2\,a\,\sqrt{b}\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-$$

$$\frac{c\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{2\,a^{3/2}}+\frac{f\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]}{2\,a^{3/4}\,b^{3/4}\,\sqrt{a+b\,x^4}}+$$

$$\left(\left(\sqrt{b}\,d-\sqrt{a}\,f\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]\right)\Bigg/$$

$$\left(4\,a^{5/4}\,b^{3/4}\,\sqrt{a+b\,x^4}\right)$$

Result (type 4, 225 leaves):

$$\frac{1}{2 a^{3/2} b \sqrt{a+b x^4}}\left(\sqrt{a}\, b\, (c+x (d+x (e+f x)))-b c \sqrt{a+b x^4}\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]+\right.$$

$$i a^{3/2} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, f \sqrt{1+\frac{b x^4}{a}}\, \text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]+$$

$$\left.\frac{b \left(\sqrt{b}\, d+i \sqrt{a}\, f\right) \sqrt{1+\frac{b x^4}{a}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, x\right],-1\right]}{\left(\frac{i \sqrt{b}}{\sqrt{a}}\right)^{3/2}}\right)$$

Problem 548: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c+d x+e x^2+f x^3}{x^2 \left(a+b x^4\right)^{3/2}}\, dx$$

Optimal (type 4, 344 leaves, 13 steps):

$$\frac{x \left(a e+a f x-b c x^2-b d x^3\right)}{2 a^2 \sqrt{a+b x^4}}+\frac{d \sqrt{a+b x^4}}{2 a^2}-\frac{c \sqrt{a+b x^4}}{a^2 x}+$$

$$\frac{3 \sqrt{b}\, c\, x \sqrt{a+b x^4}}{2 a^2 \left(\sqrt{a}+\sqrt{b}\, x^2\right)}-\frac{d\, \text{ArcTanh}\left[\frac{\sqrt{a+b x^4}}{\sqrt{a}}\right]}{2 a^{3/2}}-\frac{1}{2 a^{7/4} \sqrt{a+b x^4}}$$

$$3 b^{1/4} c \left(\sqrt{a}+\sqrt{b}\, x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\, \text{EllipticE}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]+$$

$$\left(\left(3 \sqrt{b}\, c+\sqrt{a}\, e\right) \left(\sqrt{a}+\sqrt{b}\, x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(4 a^{7/4} b^{1/4} \sqrt{a+b x^4}\right)$$

Result (type 4, 245 leaves):

$$\left(\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\left(-2\,a\,c-3\,b\,c\,x^4+a\,x\,\left(d+x\,\left(e+f\,x\right)\right)\right)-\sqrt{a}\,d\,x\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+$$
$$3\,\sqrt{a}\,\sqrt{b}\,c\,x\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-\mathrm{i}\,\sqrt{a}\,\left(-3\,\mathrm{i}\,\sqrt{b}\,c+\sqrt{a}\,e\right)$$
$$x\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\Bigg)\Bigg/\left(2\,a^2\,\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\,x\,\sqrt{a+b\,x^4}\right)$$

Problem 549: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{c+d\,x+e\,x^2+f\,x^3}{x^3\,\left(a+b\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 367 leaves, 15 steps):

$$\frac{x\,\left(a\,f-b\,c\,x-b\,d\,x^2-b\,e\,x^3\right)}{2\,a^2\,\sqrt{a+b\,x^4}}+\frac{e\,\sqrt{a+b\,x^4}}{2\,a^2}-\frac{c\,\sqrt{a+b\,x^4}}{2\,a^2\,x^2}-$$
$$\frac{d\,\sqrt{a+b\,x^4}}{a^2\,x}+\frac{3\,\sqrt{b}\,d\,x\,\sqrt{a+b\,x^4}}{2\,a^2\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{e\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{2\,a^{3/2}}-\frac{1}{2\,a^{7/4}\,\sqrt{a+b\,x^4}}$$
$$3\,b^{1/4}\,d\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]+$$
$$\left(\left(3\,\sqrt{b}\,d+\sqrt{a}\,f\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Bigg/$$
$$\left(4\,a^{7/4}\,b^{1/4}\,\sqrt{a+b\,x^4}\right)$$

Result (type 4, 259 leaves):

$$\left(-\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\left(b\,x^4\,\left(2\,c+3\,d\,x\right)+a\,\left(c+2\,d\,x-x^2\,\left(e+f\,x\right)\right)\right)+\sqrt{a}\,e\,x^2\,\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+$$
$$3\,\sqrt{a}\,\sqrt{b}\,d\,x^2\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-\mathrm{i}\,\sqrt{a}\,\left(-3\,\mathrm{i}\,\sqrt{b}\,d+\sqrt{a}\,f\right)$$
$$x^2\,\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\Bigg)\Bigg/\left(2\,a^2\,\sqrt{\frac{\mathrm{i}\sqrt{b}}{\sqrt{a}}}\,x^2\,\sqrt{a+b\,x^4}\right)$$

Problem 550: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{c+d\,x+e\,x^2+f\,x^3}{x^4\left(a+b\,x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 387 leaves, 17 steps):

$$-\frac{x\left(b\,c+b\,d\,x+b\,e\,x^2+b\,f\,x^3\right)}{2\,a^2\sqrt{a+b\,x^4}}+\frac{f\sqrt{a+b\,x^4}}{2\,a^2}-\frac{c\sqrt{a+b\,x^4}}{3\,a^2\,x^3}-\frac{d\sqrt{a+b\,x^4}}{2\,a^2\,x^2}-\frac{e\sqrt{a+b\,x^4}}{a^2\,x}+\frac{3\sqrt{b}\,e\,x\sqrt{a+b\,x^4}}{2\,a^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)}-\frac{f\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]}{2\,a^{3/2}}-\frac{1}{2\,a^{7/4}\sqrt{a+b\,x^4}}3\,b^{1/4}\,e\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]-\frac{1}{12\,a^{9/4}\sqrt{a+b\,x^4}}b^{1/4}\left(5\sqrt{b}\,c-9\sqrt{a}\,e\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 267 leaves):

$$\left(-\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\left(2\,a\,c+b\,x^4\left(5\,c+6\,d\,x+9\,e\,x^2\right)+3\,a\,x\left(d+x\left(2\,e-f\,x\right)\right)+3\sqrt{a}\,f\,x^3\sqrt{a+b\,x^4}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,x^4}}{\sqrt{a}}\right]\right)+9\sqrt{a}\,\sqrt{b}\,e\,x^3\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]-\sqrt{b}\left(-5\,i\sqrt{b}\,c+9\sqrt{a}\,e\right)x^3\sqrt{1+\frac{b\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right],-1\right]\right)\Big/\left(6\,a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x^3\sqrt{a+b\,x^4}\right)$$

Problem 557: Result more than twice size of optimal antiderivative.

$$\int \frac{81+36\,x^2+16\,x^4}{729-64\,x^6}\,dx$$

Optimal (type 3, 10 leaves, 2 steps):

$$\frac{1}{6}\,\text{ArcTanh}\left[\frac{2\,x}{3}\right]$$

Result (type 3, 21 leaves):

$$-\frac{1}{12}\,\text{Log}[3-2\,x]+\frac{1}{12}\,\text{Log}[3+2\,x]$$

Problem 567: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{81+36\,x^2+16\,x^4}{\left(729-64\,x^6\right)^2}\,dx$$

Optimal (type 3, 81 leaves, 8 steps):

$$\frac{1}{17496\,(3-2\,x)}-\frac{1}{17496\,(3+2\,x)}-\frac{\text{ArcTan}\left[\frac{3-4\,x}{3\,\sqrt{3}}\right]}{13122\,\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{3+4\,x}{3\,\sqrt{3}}\right]}{13122\,\sqrt{3}}+\frac{\text{ArcTanh}\left[\frac{2\,x}{3}\right]}{8748}$$

Result (type 3, 122 leaves):

$$\frac{1}{157464}\left(\frac{36\,x}{9-4\,x^2}+3\,\sqrt{3}\,\text{ArcTan}\left[\frac{1}{3}\left(-i+\sqrt{3}\right)x\right]+4\,i\,\sqrt{3}\,\text{ArcTanh}\left[\frac{1}{3}\left(1-i\,\sqrt{3}\right)x\right]+\right.$$

$$\left.\left(-3+\frac{2}{\sqrt{\frac{1}{6}\left(1+i\,\sqrt{3}\right)}}\right)\text{ArcTanh}\left[\frac{1}{3}\left(x+i\,\sqrt{3}\,x\right)\right]-9\,\text{Log}[3-2\,x]+9\,\text{Log}[3+2\,x]\right)$$

Problem 583: Result more than twice size of optimal antiderivative.

$$\int\frac{c+d\,x^{-1+n}}{\left(a+b\,x^n\right)^3}\,dx$$

Optimal (type 5, 46 leaves, 3 steps):

$$-\frac{d}{2\,b\,n\,\left(a+b\,x^n\right)^2}+\frac{c\,x\,\text{Hypergeometric2F1}\left[3,\frac{1}{n},1+\frac{1}{n},-\frac{b\,x^n}{a}\right]}{a^3}$$

Result (type 5, 108 leaves):

$$\left(x\left(c+d\,x^{-1+n}\right)\left(\frac{a^2\,n\,(-a\,d+b\,c\,x)}{b\,\left(a+b\,x^n\right)^2}+\frac{a\,c\,(-1+2\,n)\,x}{a+b\,x^n}+\right.\right.$$

$$\left.\left.c\left(1-3\,n+2\,n^2\right)x\,\text{Hypergeometric2F1}\left[1,\frac{1}{n},1+\frac{1}{n},-\frac{b\,x^n}{a}\right]\right)\right)\Big/\left(2\,a^3\,n^2\left(c\,x+d\,x^n\right)\right)$$

Problem 590: Result unnecessarily involves higher level functions.

$$\int \frac{1+x^3}{\left(1-x^4\right)\left(1+x^4\right)^{1/4}}\,dx$$

Optimal (type 3, 103 leaves, 10 steps):

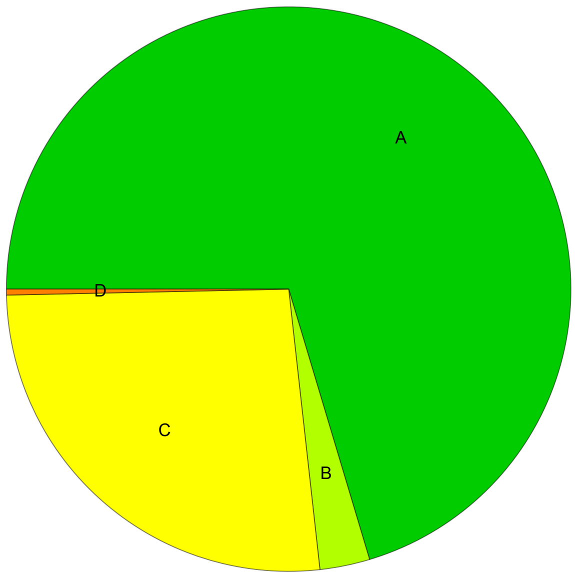
$$\frac{\text{ArcTan}\left[\frac{2^{1/4}x}{\left(1+x^4\right)^{1/4}}\right]}{2\times 2^{1/4}}-\frac{\text{ArcTan}\left[\frac{\left(1+x^4\right)^{1/4}}{2^{1/4}}\right]}{2\times 2^{1/4}}+\frac{\text{ArcTanh}\left[\frac{2^{1/4}x}{\left(1+x^4\right)^{1/4}}\right]}{2\times 2^{1/4}}+\frac{\text{ArcTanh}\left[\frac{\left(1+x^4\right)^{1/4}}{2^{1/4}}\right]}{2\times 2^{1/4}}$$

Result (type 6, 166 leaves):

$$-\left(\left(2\,x^4\,\text{AppellF1}\left[1,\frac{1}{4},1,2,-x^4,x^4\right]\right)\Big/\left(\left(-1+x^4\right)\left(1+x^4\right)^{1/4}\left(8\,\text{AppellF1}\left[1,\frac{1}{4},1,2,-x^4,x^4\right]+x^4\left(4\,\text{AppellF1}\left[2,\frac{1}{4},2,3,-x^4,x^4\right]-\text{AppellF1}\left[2,\frac{5}{4},1,3,-x^4,x^4\right]\right)\right)\right)\right)+\frac{2\,\text{ArcTan}\left[\frac{2^{1/4}x}{\left(1+x^4\right)^{1/4}}\right]-\text{Log}\left[1-\frac{2^{1/4}x}{\left(1+x^4\right)^{1/4}}\right]+\text{Log}\left[1+\frac{2^{1/4}x}{\left(1+x^4\right)^{1/4}}\right]}{4\times 2^{1/4}}$$

Summary of Integration Test Results

594 integration problems



A - 418 optimal antiderivatives

B - 17 more than twice size of optimal antiderivatives

C - 157 unnecessarily complex antiderivatives

D - 2 unable to integrate problems

E - 0 integration timeouts